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DEVELOPMENT AND APPLICATION OF A DECISION METHODOLOGY FOR THE PLANNING OF NUCLEAR RESEARCH AND DEVELOPMENT IN SAUDI ARABIA

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Development and application of a decision methodology for the planning of nuclear research and development in Saudi Arabia

by

Waleed Hussain Abulfaraj

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1. INTRODUCTION

Most developing countries are considering nuclear technology as a major element in future energy development plans. Several countries have finalized agreements to import nuclear power plants or nuclear research facilities. The role of nuclear technology differs from one country to another due to differences among their needs, aspirations, and environment. In many cases, the need for nuclear technology is dependent on national energy demands; other developing countries desire to acquire and apply the nuclear technology to fields such as medicine, agriculture, industry, food production and research in the areas of physics, engineering, biology and chemistry.

The interest of Saudi Arabia in acquiring a national research center is steadily growing. The aim and purpose of the center is:

(1) to perform basic research with the aim of studying fundamentals and basic questions in natural science and engineering;

(2) to train Saudi scientists, engineers and students on basic research and peaceful uses of nuclear energy;

(3) to provide a radioisotope production facility for use in various applications such as oil well logging, diagnosing oil pipe defects and medical

applications such as those currently existing in the King Faisal Speciality Hospital;

(4) to provide local manpower with expertise in nuclear energy and nuclear desalination technology;

(5) to provide irradiation services for local industries and hospitals and for research purposes such as activation analysis, health physics, radiology, radio-biology, and agriculture applications;

(6) to establish the means for transfer of nuclear technology from nuclear countries to the Saudi Arabian region;

(7) to have a forum for Saudi scientists to interact with other scientists and to provide opportunity for the Saudis to contribute to the state-of-the-art of peaceful applications of nuclear energy.

In this study, decision theory is employed in providing a strategy for implementation of a Saudi nuclear research center (SNRC) including selection of a research reactor facility and prioritization of R&D programs. The research plan of the center would depend on the type of facility and on the flexibility of such a facility. The research program has two parts: basic research, and mission-oriented research and development. The type and magnitude of the basic research component would depend on available financing and the interest of individuals to be on the staff of the center.

The mission-oriented component has to be directed to the immediate needs of the country. Thus, in selection of the facility, mission-oriented R&D has to be considered first.

A methodology is developed here to select a nuclear research facility and to provide an R&D strategy for the SNRC. The decision methodology used here is developed, bearing in mind the need to provide the decision makers in the SNRC with a formal approach for application in all decision problems related to the SNRC. Examples of such problems are siting of the research center, selection of experiments, and prioritization of R&D programs within a constraint of limited yearly budget.

The purpose of this study is the development of a methodology appropriate to making decisions on issues related to peaceful uses of nuclear energy and the establishment of an SNRC with its facilities and programs; development of a methodology which is viable, practical and simple for use by decision makers in the SNRC to handle yearly and daily decision problems; and application of the developed methodology to the selection and siting of the nuclear research reactor facility and to prioritization of R&D programs for the SNRC.

As a first step, a review of available nuclear research reactors is presented to identify factors to be considered in selecting the preferred facility of the SNRC. Chapter

2 covers the basic concept, characteristics, material used, safety considerations, experimental facilities available, and common terms for each research reactor type. Also, a very brief description is given for other reactor types which are not appropriate for this study because of their complicated design, uses for specific purposes, excessive size, low power density, maintenance problems, cost, and/or uncertainty of long-term utilization.

To select a suitable decision methodology for the present case, a critical survey of available decision methods is conducted in Chapter 3. These methods have been developed and applied with varying degrees of success to many diverse areas of interest in the field of economics, business, management, war games and strategies, applied statistics, and operation research.

Based on the critical survey, the multiattribute utility function is selected to evaluate the various research reactor facilities. Among the points which are in favor of this approach are:

(1) Consideration of tangibles and monetized factors as well as intangibles;

(2) Ability to reduce multidimensional factors into one function useful for ranking;

(3) Accommodation for decision makers' (experts, administrators, legislators, public groups)

preferences;

(4) Amenability to analytic solutions and computer programming;

(5) Flexibility in iterative applications and in application in all study areas of the nuclear research center;

(6) Applicability to various problems similar in nature to the present study;

(7) Simplicity of input information and directness of the output results.

The multiattribute utility theory has been applied in assessing inventory ordering policy for a hospital blood bank [1], in assessing the safety of landing aircraft in various weather conditions [2], in the selection of an appropriate program for a forest pest problem [3], in selection of management policies [4], and in many other fields [5-10]. In Chapter 4, a historical background, the fundamentals of utility theory and the procedure for assessing utility functions are presented. The concept of multiattribute utility theory is also introduced with focusing on a two attribute problem.

In Chapter 5, the objectives of the SNRC are defined and structured into definite subobjectives for which attributes and subattributes are easily derived and defined. The set of the center's objectives is economic, technological, and safety feasibility as well as capability of providing most anticipated functions and services, and compatibility with the local environment. Definitions of each category, attribute, and subattributes are developed to assure the uniqueness of each, to avoid confused interpretation, and to eliminate double counting.

Four alternative research reactor facilities have been chosen, namely the University of Michigan Ford Nuclear Reactor (FNR), the Massachusetts Institute of Technology Reactor (MITR), the Georgia Institute of Technology Research Reactor (GTRR), and the University of Wisconsin Nuclear Reactor (UWNR). These are pool, light water tank, heavy water tank and TRIGA reactors, respectively. The values of the developed categories, attributes, and subattributes corresponding to the level of impact of each alternative are determined in Chapter 6 according to available data and sources.

In Chapter 7, adapting acceptable ranges, verifying preferential and utility independence, constructing utility curves through a lottery procedure, and assessing the scaling constants have been conducted to apply the multiattribute utility functions which are then used to select among the four alternatives.

A computer program has been developed to assist in performing a utility analysis. The program calculates the coefficients for each measure which represent the best fit to an exponential curve, the value of the scaling constants, and the utility for each alternative. The computer program listing is given in Appendix A.

Also based on the critical survey of decisions methods in Chapter 3, fuzzy set theory is chosen to handle decisions on site selection for the nuclear research center in Saudi Arabia. The approach allows for accommodation of imprecision in evaluation of the factors impacting site selection; such as the site geology, hydrology, seismology, topography, meteorology, and availability of cooling water, services and transportation. Two specific sites are considered, one on the East Coast near Dhahran and the second near Jeddah on the West Coast. The results are reported in Chapter 8. Conclusions drawn from the work presented here are given in Chapter 9.

Procurement and siting decisions in Saudi Arabia for equipment of this sort are based on the demonstrated performance of operating units. Equipment ordered will duplicate units in existence and may include peripheral components whose function and utility have already been demonstrated. Certain procedures discussed in detail in this dissertation reflect this modus operandi. For example, performance specifications are not written to meet the purchaser's needs directly, but to agree with the way in which satisfactory existing units operate.

2. REVIEW OF RESEARCH REACTOR SYSTEMS

2.1. Introduction

In order to evaluate how different types of research reactors meet the selection criteria, the basic characteristics of available research reactors are reviewed. A research reactor means a reactor built to provide a strong source of neutron and gamma rays for training of nuclear engineers, for engineering and industrial research, fundamental pure scientific studies in the fields of physics, chemistry, metallurgy, biology, medical studies and for agricultural research [11,12].

A research reactor is generally classified according to the type and arrangement of fuel, moderator and coolant used and the neutron flux or thermal power [13]. Zero power and low flux reactors generally operate at power levels below one MW (few watts up to near 1 MW), and are primarily used for training and basic research experiments of reactor physics and applied sciences. Such reactors do not require much cooling, and the operation and maintenance required is nominal [14]. Medium flux reactors operate at power levels between one and five MW. This permits a reasonable number of experiments in practically all fields of nuclear science and technology. High flux reactors generally operate above five MW power level. These reactors are usually intended for carrying out specific, specialized programs such as material testing, fuel element technology and corrosicn studies [15].

In the following section, a description is given of the various types of research reactors, their characteristics, basic concepts, safety consideration and some common terms. Also, the section covers design features, fuel and moderating, material used, and experimental facilities available.

2.2. Swimming Pool Type

This type of reactor is one of the first and most widely used. The first generation of these reactors was designed and constructed as early as 1950 at Oak Ridge National Laboratory for operation at a power of 10 MW. The reactor is called the "swimming pool" since its core is suspended in a large volume of water. The core is supported by an aluminum structure which is mounted vertically on a steel bridge that spans the pool. By this arrangement, the reactor core can be moved to various positions. Figure 2.1 illustrates the design features of this type of reactor.

The main advantage of such reactors is the flexibility of the core. Experiments can be set up or modified at one location of the pool, while other experiments are being carried out in another area. One end of the pool may be shaped to accommodate beam tubes and thermal columns. Some designs provide experimental facilities at both ends of the pool.

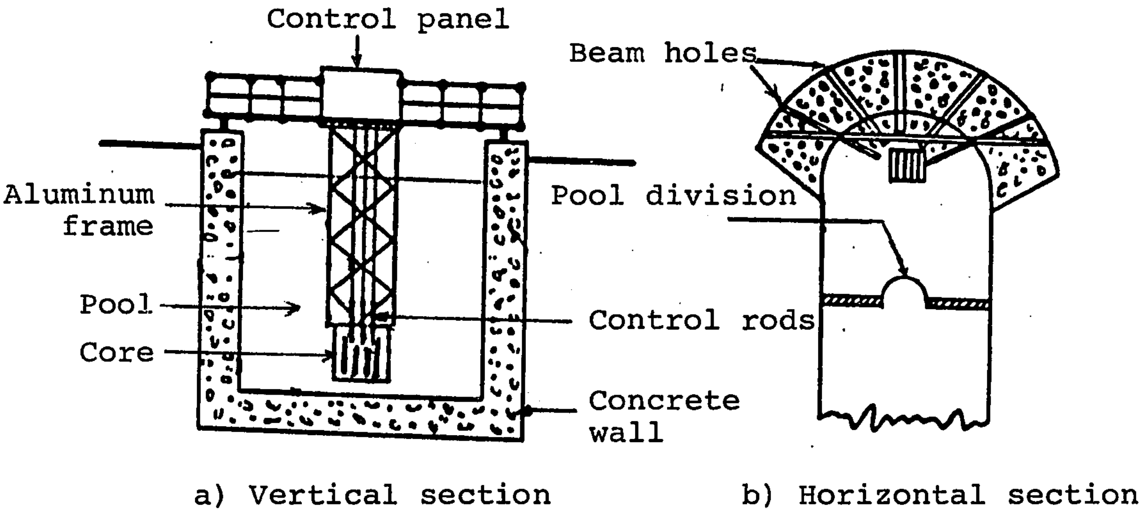


Figure 2.1. Vertical and horizontal sections of swimming pool type reactors

Pool reactor makes ingenious use of ordinary water as core coolant, moderator, reflector, and also as a shielding material in some designs. The fuel of the pool type reactors is highly enriched plate type uranium-aluminum fuel elements. This type is often called MTR-type fuel elements [12].

These reactors can operate at a power level as high as 10 MW. Normally forced cooling is required above 10 MW. Two main factors which limit the upper power level of such reactors are:

(1) the rate at which heat can be removed from fuel elements by free or forced convection; and

(2) the level of radioactivity acceptable in the pool water.

Another type of pool reactor, known as Aquarium Reactor, is slightly different from the pool reactor, but its essential operation is the same and is shown in Figure 2.2. The main features of such reactors are:

(1) the core of the reactor has a fixed position and is supported from the bottom rather than from the top in a small diameter tank;

(2) free convective cooling only;

(3) beam holes generally not present; and

(4) reflector elements are generally present around the core [16].

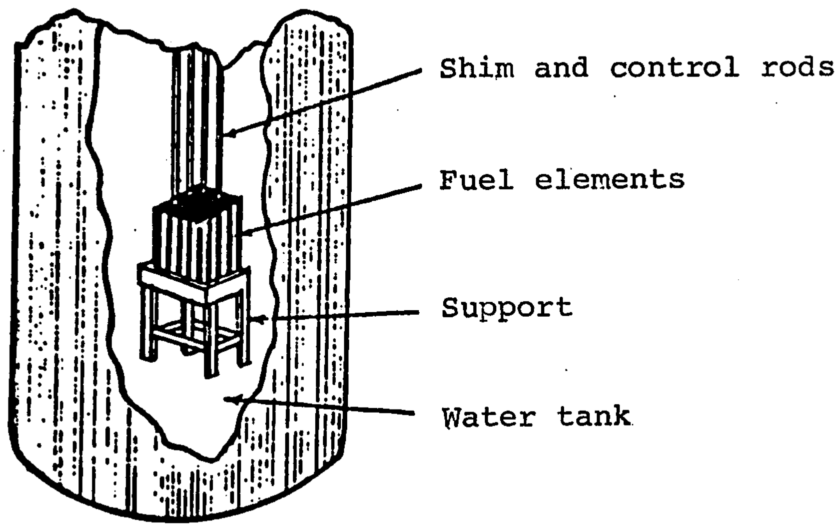


Figure 2.2. Aquarium reactor

2.3. Tank Type Reactors

The following is a very general classification of tank type reactors which is further divided into various types. The main features of this class of reactor types are given as follows:

(1) The fuel elements, coolant, and moderator are contained in a metal tank. The size of the tank depends on the material used, intended use, and the power of the reactor. The tank may be open or closed at the top.

(2) Generally graphite, beryllium or beryllium oxide is used as a reflector. In some reactors, two or more layers of different reflector materials have been employed.

(3) The power range of these reactors are from a few kw to many MW.

This class of reactors may be further subdivided in many types depending upon the fuel, moderator and reflector options:

(1) fuel-enriched uranium, natural uranium;

(2) moderator and coolant--D_2O and H_2O; and

(3) reflector--D_2O, H_2O, Be, BeO, and C.

Various combinations of fuel, moderator, coolant, and reflector materials are used. More prominent types of tank reactor are summarized in Table 2.1.

Table 2.1. Types of tank research reactors

Fuel	Moderator	Coolant	Reflector
Enriched uranium	D_2O	D_2O	D_2O/graphite
Natural uranium	D_2O	D_2O	Graphite, Be or BeO
Enriched uranium	H_2O	H_2O	Be, BeO or graphite
Enriched uranium	Graphite/H_2O	H_2O	Graphite
Natural uranium	D_2O	CO_2	Graphite

As indicated earlier, tank reactors may either be "open-tank" or "closed-tank" type. The first of these generally operates below 5 MW, while the second can go to much higher levels. The main advantage of these reactors is that a fixed core provides more space for beam tubes and other experimental facilities, which can be used simultaneously. Access to the core from the reactor top is also possible for in-core experiments, even though it is not as easy as in the pool type reactors. The installation is generally much more compact. It should be pointed out here that all reactors using heavy water are closed tank types, mainly due to associated tritium radioactivity problems, and to avoid a decrease in the isotopic purity of D_2O by contamination by H_2O [17]. A side view of a typical tank type reactor is given in Figure 2.3.

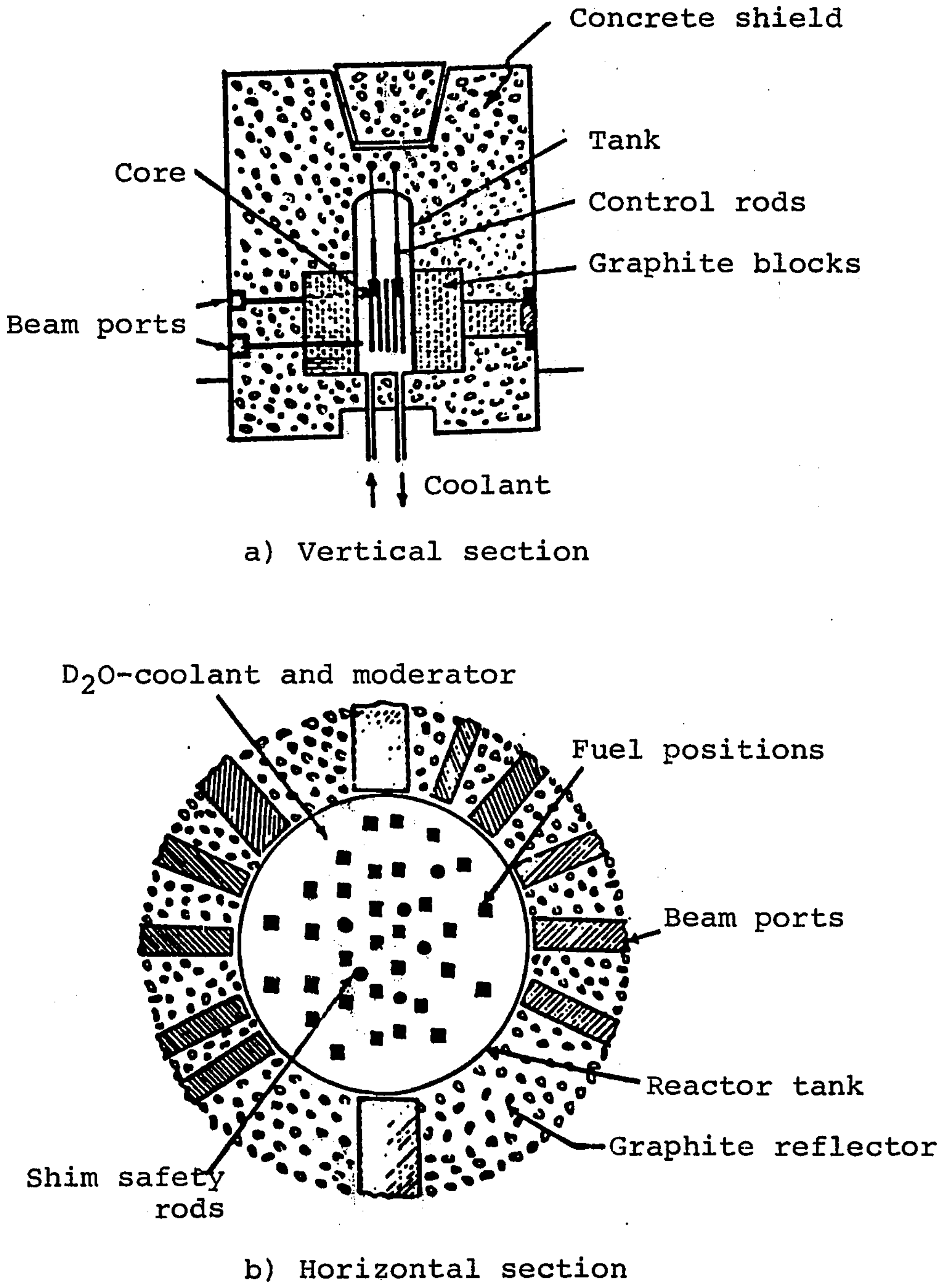


Figure 2.3. Tank type reactor

An example of this type of reactor is the MIT reactor whose core tank is four feet in diameter and approximately 7 feet high. The fuel elements are U-Al MTR type plates. This reactor is H_2O cooled and moderated with graphite and D_2O used as reflectors. The number of experimental ports and other facilities in such reactors can be made greater than in any other type of reactor (Figure 2.3).

2.4. Pulsed Reactors

The two main features of this type of reactor are:

(1) Moderator material admixed with enriched fuel in metallic alloy (enriched uranium as fuel with zirconium hydride as moderator).

(2) Taking advantage of the Doppler broadening effect for regulation.

The pulsing can be done at intervals of as little as 5 minutes by the rapid ejection of a pulse control rod from the core. This makes the reactor supercritical, even prompt-critical. The flux and power level then immediately increase at an exceedingly rapid rate reaching their peak within a matter of a few milliseconds. The pulse, however, promptly quenches itself with the control rods out and without any external action. The hydrogen moderator in the hydrided U-Zr alloy fuel elements promptly rises to high temperature (of the order of 500^{o}C) as a result of rapid fission heating.

At such temperatures, hydrogen can only moderate neutrons to kinetic energies equivalent to temperature of 500°C. Since the U-235 fission cross section is very low at these energies, the system becomes subcritical. As the power level declines and the fuel element temperatures fall, the core power becomes steady at a low level.

An example of this type of reactor is TRIGA Mark I, II, and III. TRIGA Mark I is capable of operation at power levels up to 10 kw while Mark II and Mark III can operate at steady state levels of 250 kw and 1 MW, respectively. The pulsing capability of TRIGA reactors ranges from 250 MW to 2,000 MW, giving neutron levels of 10^{16} neutrons/cm^2-sec or higher. These flux levels are three to four orders of magnitude higher than that available in the highest flux university research reactors [18].

This type of reactor is generally used for educational and training purposes, production of short-lived isotopes, radiation effects and activation analysis. The limitations of this type are that any power exceeding the continuous operating power will result in loss of reactivity due to temperature rise and the reactor will shut itself down. Furthermore, at higher power polyethylene undergoes considerable radiation damage and hence the core-life is reduced.

2.5. Other Types of Reactors

Other types of research reactors are not appropriate and beyond the scope of this study because of one or more of the following reasons:

(1) complicated design features;

(2) used for specific purposes;

(3) excessive size;

(4) low power density;

(5) maintenance problems;

(6) cost;

(7) uncertainty of long term utilization.

A very brief description, however, is given for these types of research reactors to complete the general understanding of research reactors.

2.5.1 Graphite moderated reactors

Graphite moderated reactors are the largest in size. The first reactor of this type, the BP-1, was operated by Enrico Fermi at nominal power level of 2 W. The graphite blocks occupied a volume 6.68 m x 6.84 m x 6.84 m while 23.9 tons of uranium, in the form of cylindrical rods, were arranged in a stack of 14,000 graphite blocks, without any mechanical connection [19]. The reactors of this type are cooled by CO_2 or other gases. The power range of these reactors varies from a few watts to many megawatts. Natural or slightly enriched uranium is used as a fuel. Figure 2.4 shows such a reactor.

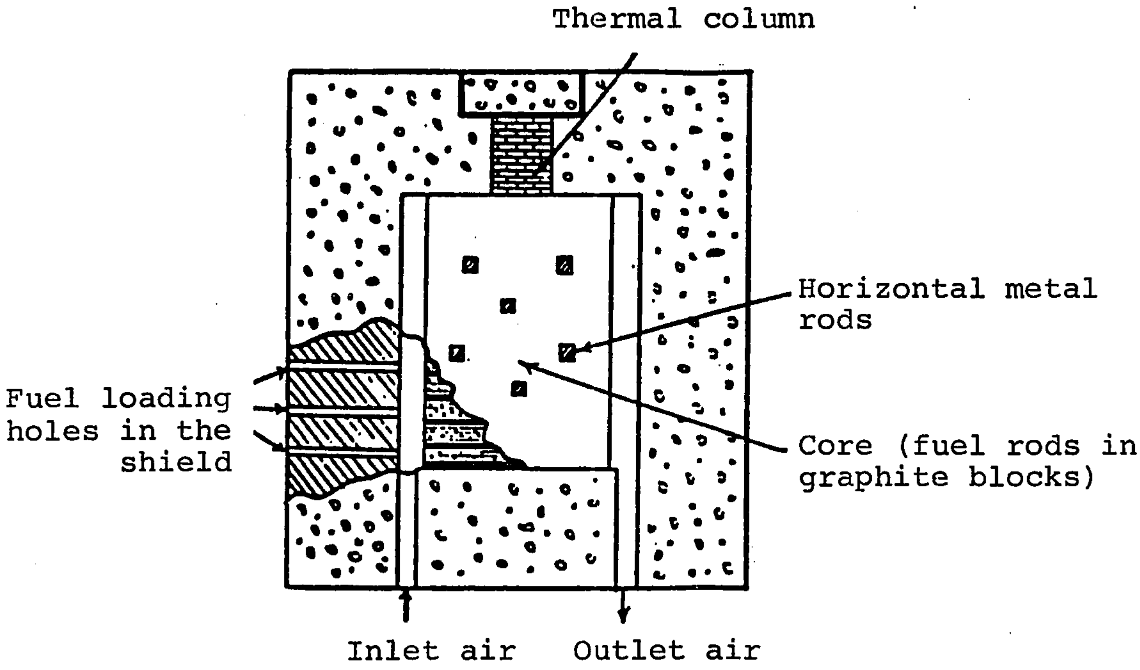


Figure 2.4. Graphite moderated reactor

2.5.2 Fast research reactors

In fast research reactors, only fissionable material is used. The fuel usually consists of highly enriched uranium or plutonium and is cooled either by liquid metals or gases. These reactors are used for a few and specific purposes such as breeding reactors, instrument testing and simulation of weapon [20].

2.5.3 Flux trap reactors

This type is generally used for obtaining high thermal neutron fluxes. The reactor consists of a shell containing fissionable material surrounding a central moderator, which forms the "flux-trap". Fission neutrons from the shell slow down in the moderator. Since the moderator has a low absorption cross section, the thermal flux rises to a level such that the radial gradient provides the necessary flow outward. The flux trap is sometimes referred to as an island or internal thermal column. In the central moderator column, circular cross section vertical holes are provided in which samples to be irradiated can be inserted (see Figure 2.5). These reactors are usually designed for thermal powers of few hundred kw and used for production of high transuranic elements, neutron cross section measurement and neutron diffraction studies.

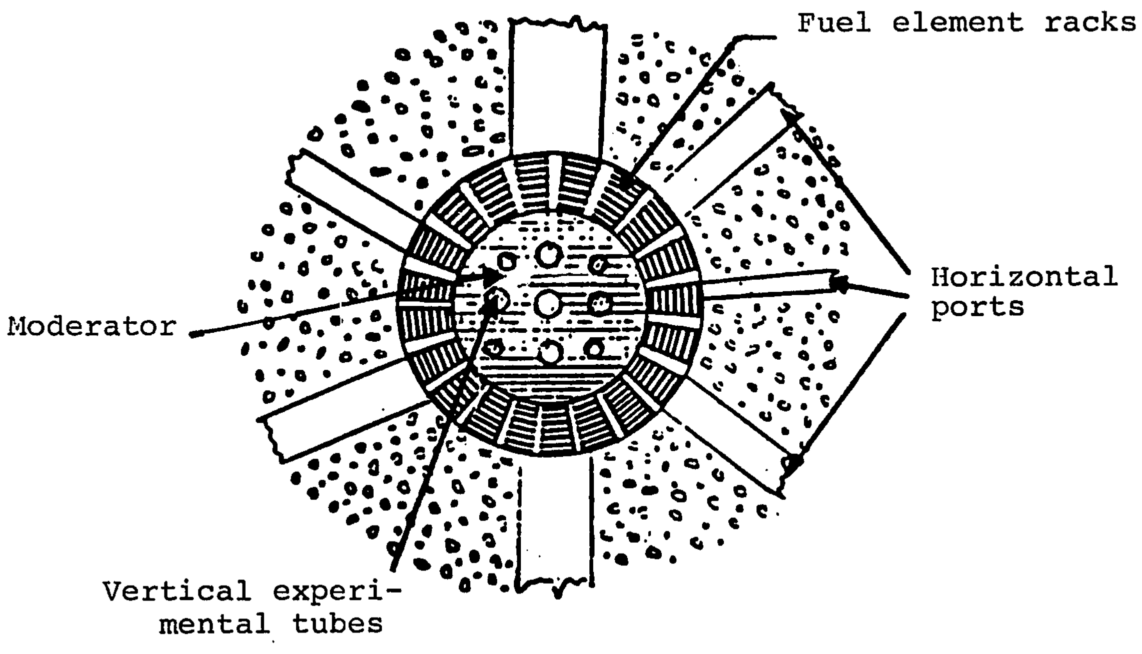


Figure 2.5. Flux trap reactor

2.5.4 Liquid homogeneous reactors

Liquid homogeneous reactors are frequently called "water boiler reactor or solution type" (Figure 2.6). The core of these types of reactors usually is a stainless steel sphere which contains the fissionable material and moderator in a homogeneous solution of uranium salt; e.g., uranium sulfate in light or heavy water solution. The fuel solution is cooled by water circulating through the coils inside the core. Normally this cooling keeps the solution temperature below 80^{o}C and no actual boiling occurs. The name water boiler comes from the bubbling which results from the formation of hydrogen and oxygen produced by decomposition of the water by fission fragments. There are several openings into the sphere for filling, control rod insertion and venting the fission gases and H_2 and O_2 formed in the core. The

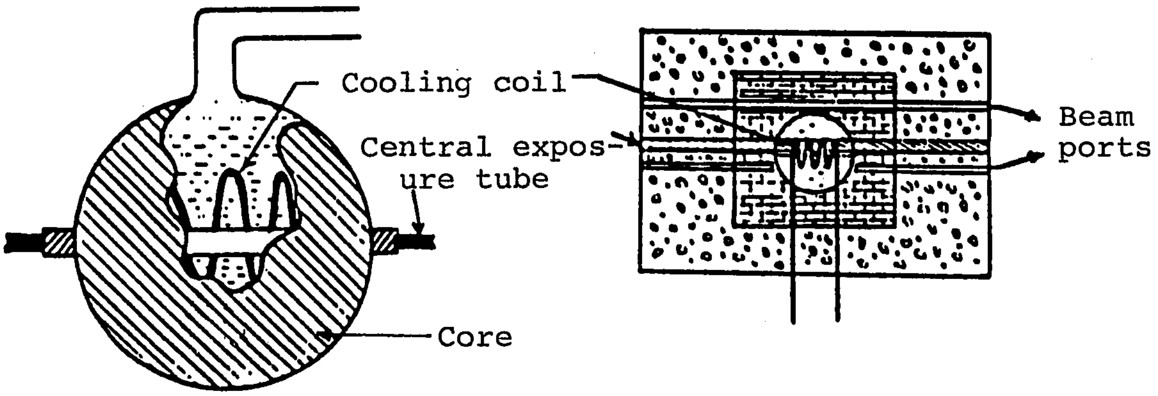


Figure 2.6. Liquid homogeneous reactor

core is surrounded by a suitable reflector like graphite, beryllium or beryllium oxide. A concrete shield around the reflector protects personnel from radiation. Generally, the fuel is stored external to the core when the reactor is not operating. In order to start the reactor, the solution is pumped into the core and the position of the control rod is adjusted to regulate the reaction [16].

This type of reactor generally falls into three main power categories. The lowest power size is designed for training purposes. The medium-sized reactors operate at power levels of about 1 kw while the larger ones operate around 50 kw. The main advantages of this type of reactor are:

(1) simplest fuel arrangements of all reactor types requiring no fuel fabrication;

(2) continuous refueling and purification possible;

(3) inherent safety due to negative temperature and, therefore, they are well suited for training, even at very densely populated locations.

The disadvantages of these reactors are:

(1) low neutron flux;

(2) possibility of corrosion;

(3) fuel leakage.

3. DECISION THEORY

3.1. Introduction

Since man has existed, he has been faced with the need for making decisions in choosing one of several possible alternatives. The earliest decision theorists were concerned with gambling decisions as advisers to gamblers in the French court [21]. The theory of decision developed from these beginnings. It was not until the early 20th century that developments in decision theory gained a noticeable momentum [22]. A broad spectrum of concepts and techniques of decision theory has been developed and applied with varying degrees of success to many diverse areas of interest in the field of economics, business, management, war games and strategies, applied statistics, operations research, etc. In this chapter, critical surveys of available R&D decision methods are discussed in detail.

3.2. Critical Survey of Available R&D Decision Methods Under Uncertainties for Selection Among Few Alternatives

Several methods have been proposed for the development of R&D strategies. Most of those were developed in the field of econometrics and operation research to improve government efficiency and reduce unwarranted spending [23]. The presentation is often sketchy and falls more or less into the realm of politicizing rather than planning and policymaking. In a businesslike manner, many of the quantitative approaches are concerned with minimizing a cost function of one type or another, such as C(M,T) where M refers to R&D monetary expenditures on a given project and T refers to the time required to execute that project [23]. The cost function may be expressed as a linear combination,

$$C(M,T) = SM + h(T)$$

where S is a non-negative constant and h(T) is some non-decreasing function of T.

3.2.1 The deterministic approach

A deterministic approach which has been frequently applied in R&D planning is simply to ignore uncertainty or risk [24]. This approach stems from the fact that both in the theoretical analysis and operation of actual systems it is often expedient to act as if consequences of various decisions can be predicted accurately. The approach has evolved considerably from being a traditional method to one that is being applied to contemporary R&D problems. In the traditional sense, the term "uncertainty" has been used to describe situations whose outcomes are based on unknown probability distributions. If a probability distribution, based on objective probabilities or relative frequences of the outcomes, is known, the situation is a "risky" one [24]. In modern decision analysis, personal (or subjective) probabilities are acceptable and hence there is no distinction between risk and uncertainty. In the application of the deterministic method, the planner uses "best estimates" (or most likely values) of the project variables (such as cost and return) in formulating a "measure of merit" which is envisioned as an approximation of the "most likely" outcome of a given design alternative. Measures of merit are, for example, benefit-cost ratio, net benefit or other performance criterion. The approach is clearly less tedious than the more advanced quantitative methods used in investment analysis. However, there is only a small likelihood that the single-valued estimates utilized in the analysis will prove accurate, and there is no provision for accommodating for the likely variation about an expected outcome of a given project. Nevertheless, there are some conditions under which assumed certainty can provide the correct approach [25]. Those conditions follow from the "certainty equivalence theorem" [25]. The deterministic approach fails to explicitly consider factors such as supply and demand uncertainties which surround each project alternative.

3.2.2 Benefit-cost approaches

3.2.2.1 Ratio The benefit-cost ratio is widely employed in selection of R&D programs. In this method, the ratio must exceed unity for a project or design to be justified. To select between alternate strategies, the one with the highest ratio is the best. In some situations, this approach may lead to false conclusions. For example, two projects may have the same benefit-cost ratio based on the most likely values of each component, but one project may be more "risky" in the sense that it is much more vulnerable to possible adverse conditions. Thus, additional information must be incorporated in the decision process before rational choices can be made [23].

3.2.2.2 Difference A method which can be shown to be logically erroneous, if no risk is included, is the benefit-cost difference. The major practical difficulties in using any of the benefit-cost methods or a combination of both lie in the tendency toward optimistic inflation of the benefit and the underestimation of cost.

3.2.2.3 Two-valued preference Another approach which is based on the use of two-valued preferences can be employed when certainty is assumed for expediency. Once a world of complete certainty is assumed, the developer's task is well-defined, since a prior knowledge exists concerning in what region of the performance space the developed items lie for a given approach to development. Obviously, if one defines an ordinal cost function $C(M,T)$ for all non-negative values of M and T, then the alternative with the smaller cost function is the most preferred, since

$$C(m_1,t_1) < C(m_2,t_2)$$

if (m_1, t_1) is preferred to (m_2, t_2),

where m and t are specific values or outcomes of variables or events M and T, respectively. In the presence of a number of alternative approaches, the developer can rank the alternative combinations of time and money (or any other two variables) that satisfy some predetermined condition.

Since the developer knows the time and money each approach would require if it were pursued to completion, the cost function $C(M,T)$ can compactly represent the preferences for a given item and a given satisfactory region of its performance space. Actually, the developer prefers to commit less money to more, and less time to more, but is willing to trade time and money against each other. Consequently, the approach having the smallest value of completion cost $C(m,t)$ is the one to be picked and pursued by itself. Such certainty method can be expanded to encompass a host of variables by defining a cost function $C(M,t,\Pi)$ where Π is a vector that implicitly includes an arbitrary number of variables. In this case, the analysis can only be completed if a form of C is depicted based on experience or on some empirical rules. The validity of the approach depends on whether or not the developer accepts the world of complete certainty for expediency.

3.2.2.4 Cost effectiveness In this approach, multi-valued benefits could be used and budget constraints may be introduced. For parallel programs, the sum of proposed expenditures has to be less or equal to the ceiling on the budget. An aspiration level of benefits is set as a priority to program selection. Then each program is evaluated to find whether the benefits gained would be less than or equal to the aspiration levels. Meeting those multi-benefit levels can be used as criteria for program/proposal ranking. The difficulties in applying this approach is in determining aspiration levels, accommodating for uncertainties, conducting tradeoffs, and allowing for group decisions.

3.2.2.5 Value-impact The primary objective for a value-impact analysis of a research program is to compare the technical merits as well as to address the politico-social aspects of the regulatory process. An integral part of the value-impact analysis is the identification of qualitative values associated with a research program. Quantitative values in terms of program costs can be derived from the criteria established for a program. The overall impact assessment allows a decision-maker to make a balanced value judgment which is meaningful.

3.2.2.6 Accommodation for uncertainties Uncertainty is explicitly recognized typically by such deterministic methods as conservative estimation of the variables involved;

for example, (a) returns and costs, (b) the addition of a premium to the discount rate (see p. 29), and (c) the economic life of a project. Conservative estimation approaches have been specifically employed by federal agencies [26].

3.2.2.6.1 Adjustment of returns and costs In the technique of adjusting returns and costs, a benefit-cost ratio may be employed to rank alternative R&D strategies. Conservative adjustments are used in such a way that benefit estimates are to be reduced and cost estimates increased in some proportion. This is due to the lack of confidence in the expected or most likely values. The main drawback in the conservative method is the fact that there is a small likelihood that all estimates will accidentally be in error in the same sense. Consequently, this method of compensating for uncertainties obscures important information. For example, in examining a benefit-cost ratio based on conservative estimates, there is no indication of the degree of conservatism and there is no assurance that a higher benefit-cost ratio will result. Moreover, conservatism displays an aversion to uncertainty which is not necessarily the best attitude of a decisionmaker. Generally, conservatism in benefit and cost estimates is not regarded as an appropriate means of counteracting uncertainty in "expected values" [27]. However, conservatism may be often used as an appropriate countermeasure for the invariably optimistic bias of the decision maker in estimating benefits and costs. Nevertheless, biased estimates is an issue separate from the underlying uncertainty of such estimates.

3.2.2.6.2 Discount rate The technique of adjusting the "discount rate" is a more sophisticated approach to allow for uncertainty. The discount rate is a method formally used in development planning and is based on the economic evaluation of alternative projects to select the item with the lowest present value. The method implicitly takes into account the time necessary for completion of the projects. The discount rate, i, may be adjusted to allow for uncertainty by breaking it into two components, that is,

$$i = i_t + i_r$$

where subscripts t and r refer to time and risk components of the discount rate, respectively. The less certain the outcome of a given R&D strategy, the higher the value of i_r and hence i for a given i_t. Thus,

$$i_r \simeq r$$

where r is an index of riskiness. Consequently, the higher the risk discount rate associated with a given strategy, the more risky the strategy is. The main disadvantages of adjusting the discount rate are:

(1) It represents an ordinary aversion to uncertainty

as the technique of adjusting cost and benefit estimates.

(2) Variation in the discount rate has very little effect on the outcome of a project having a short design life.

(3) When a particular design is being evaluated from the standpoint of cost alone to meet more specified demand, increasing the discount rate to reflect cost uncertainty will have an effect opposite to what is intended, that is a higher discount rate will lower the present value of cost, with the result that higher risk alternatives are favored.

(4) Two risk premiums must be considered; one is the "expected-value adjustment" premium which is equivalent (in the case of government R&D projects) to assigning a higher discount rate than the government borrowing rate. The second is "risk-aversion" premium. The government is also capable of pooling risks and hence the technique could lead to favoring more risky strategies.

3.2.2.6.3 Economic life The conservative technique of adjusting the economic life of projects under examination also fails to provide a reasonable approach to uncertainty accommodation. This is especially true in situations where a large number of mutually independent alternate strategies contribute to aggregated objectives.

3.2.2.7 Other cost-benefit techniques Several other techniques, not necessarily conservative, have been developed in cost-benefit analysis (CBA) which has practically evolved as a discipline by itself. Application of the CBA to evaluation of public projects dates back to the work of Depuit in 1844 wherein the concept of net social benefits of projects was initiated. The ideas presented in this early work on CBA have been generalized and developed further in specific applications. The CBA field gained momentum after the U.S. Flood Control Act of 1936 which declares that "benefits to whomsoever they may accrue of Federal projects should exceed costs." Consequently, several government documents have been issued in an attempt to standardize the CBA methodology. Most of the documents were released in the context of water resources developments. This is why many of the methods developed at that time were a variation of the riskless conservative approach. While Federal efforts were directed toward revisions of that approach in practice, a firm theoretical base of CBA was being constructed in various fields [28].

3.2.2.8 Intangibles and incommensurables In order to accommodate for non-monetary factors in the CBA of alternate projects, several provisions have been made. For example, market prices do not always accurately reflect social value and hence "shadow pricing" is used. A shadow price may be defined as a value associated with a unit of some good which indicates how much some specified index of performance can be increased or decreased by the use or loss of the marginal unit of that commodity. The use of such a factor is promoted by the attempt to present all factors and outcomes in terms of a monetary value. Nevertheless, there are still some incommensurables and intangibles which are not measurable in even their own terms. To overcome that difficulty, balance sheets have been developed for cost-benefit analysis which are divided into two parts: monetized effects and non-monetized effects. In the latter part, incommensurables and intangibles which are not susceptible to quantification or monetization are displayed. However, there is an argument against considering incommensurables without assigning a shadow price to them since in principle every outcome has a social opportunity cost [27].

3.2.3 Sensitivity analysis

Sensitivity analysis is another traditional approach which is devised to help recognize explicitly some of the uncertainties in decision-making. In its typical form, sensitivity analysis is employed to supplement certainty approaches. The sensitivity of outcomes is measured by varying the values of individual elements one at a time and noting the effect on the performance criteria (measure of merit). This method isolates the outcome effect of overestimating or underestimating an element's value and thus suggests the relative importance of accurately estimating each element. However, consideration of each single element alone disregards the fact that all elements will vary somewhat from their estimated values: the actual outcome(s) will be the result of a combination of estimating errors, not the error of just one element.

A natural extension of this approach is to alter the values of several elements (rather than a single element) simultaneously. This extension is frequently referred to as the <u>multiple-case approach</u> and typically takes the form of determining the effect of optimistic, most likely, and pessimistic estimates of all relevant elements of factors such as supply and demand. The method results in the establishment of limits on the probable outcomes and the most likely outcome for each possible decision.

These two forms of sensitivity analysis, although they are more effective tools in the face of uncertainty than the assumed certainty approach or the adjustment approaches, lack both conciseness and comprehensiveness. Knowledge of how much variation is required in a particular element in order to reverse a decision based on best-estimate values, or knowledge of the range of possible outcomes for a given decision, is of limited value to decision makers if they do not take the next step of estimating the relative frequencies (probabilities) with which these outcomes occur.

3.2.4 R&D strategy

The traditional methods have features which are useful for dealing with some aspects of uncertainty; all are inadequate in several respects. Decision-making can be improved by a more systematic, complete, and explicit treatment of uncertainty, using certain aspects of decision theory and what has come to be known as research and development (R&D) strategy [29].

3.2.4.1 Probability theory and computer simulation In recent years, considerable effort in the fields of management, econometrics, and operational research has been devoted to developing improved techniques of analysis under conditions of uncertainty. Most of these techniques involve the direct utilization of probability theory and computer analysis.

As previously noted, the likelihood that all "best estimates" in an analysis will prove to be exactly accurate is extremely small. Since it is the joint effect of variation in all attributes that will determine a decision's outcome, an analysis technique is required that can allow for this more complex variation. Such a technique must explicitly employ probability theory. Operationally, this involves not only estimating a range of values for each of the most important factors affecting a decision's outcome, but establishing probabilities of occurrence for each of the intervals within that range. The question of dependence or correlation among factors must also be recognized.

Seldom is there sufficient evidence to estimate probability distributions entirely from historical records, experiments, econometric analysis, or other frequency evidence. In some situations, frequency evidence providing the likelihood of occurrence for some factors does exist and is readily available. More often, objective data must be supplemented with expert judgment in order to determine the desired probability distributions. When subjective elements enter the analysis, we obtain "subjective" rather than "objective" probability distributions for the various factors. These probability distributions may be based, of course, on "more" or "less" frequency evidence and in this sense might be labeled "more" or "less" subjective. The amount of frequency evidence that ought to be <u>collated</u> and utilized in the analysis is itself a decision problem. For decision purposes, however, it makes little difference whether an outcome is "known" in terms of a more subjective or more objective probability distribution [30].

This approach differs from the traditional approach in that judgmental elements explicitly enter the underlying assumptions rather than being applied to the final results of the analysis. The subjective probabilities reflect the degree of confidence which an investigator has in the estimates that fall in his area of expertise. No matter how "objective" or "subjective" these probability distributions might be, they illustrate precisely what is unknown, as well as what is known regarding a factor's future value. Further, based on the best information available, the degree of imperfect knowledge is quantified in the form of probability statements. Such probabilities are introduced in an attempt to "represent ignorance." In contrast to those approaches which consider only the "best estimates," the use of these probability distributions allows a more complete analysis of each decision alternative. In conjunction with the computer, which is employed to enumerate the probable performance outcomes resulting from the combinations of the relevant factors, this approach gives the decision makers a more precise assessment of the risk that may be associated with each decision alternative.

3.2.4.2 Group/individual preferences The derivation of indifference curves or preference functions can be easily done at the level of the individual decision-maker. At a community level, the preference functions can be approximated by the legislative process. At a level of groups of decision-makers, some prescribed processes have been developed. The preference functions can be easily synthesized over multiple attributes, including both tangible and intangible factors. An alternative approach, which avoids explicit derivation of preference functions (utility functions or indifference curves), is simply to present the "efficiency frontier" to the relevant decision-making group and let it make a direct choice. By making the risks explicit, the risk preferences of the decision-makers are called into play and they "reveal" their preferences by their decisions [6].

3.2.4.3 Sequential decisions R&D program planners recognize that current decisions are likely to have consequences that extend over a considerable period of time. The consequences of such decisions are not a single outcome, but rather a sequence of outcomes. While sequential aspects of decision-making are recognized, for example when the construction of a system is spread over a considerable period of time, the question of timing is usually treated within an environment of certainty.

Marglin has treated the case of sequential decision-making under certainty rather completely with several important results. He demonstrates analytically, for example, that the economic merit of a project can be improved by postponing construction and that the postponement may change the measure of merit for a particular project from an undesirable figure for construction today to a desirable value for construction at some future date. Arrow, in a generalization of some of Marglin's work, has demonstrated that in some cases optimal timing of each project or component increment can be achieved by scheduling construction the first time the project reveals a positive present value. Manne, in still a more general framework, has shown that optimal construction time is determined by balancing interest costs, loss of benefits, and economies of scale. In fact, if substantial economies of scale exist, it may be optimal to build ahead of benefits [31, 32].

The literature mentioned recognized the possibility of accumulating additional information through time so that decisions can be revised sequentially. However, it does not explicitly introduce uncertainty in the form of probability distributions by which decision strategies could be formulated. Moreover, it does not explicitly introduce the possibility of allocating available resources in such a way as to reduce or resolve uncertainties. To partially resolve uncertainty, decision-makers may revise probabilities on the basis of new information by utilizing Bayesian analysis. Formally, this involves converting <u>prior</u> probabilities to <u>posterior</u> probabilities.

3.2.4.4 <u>Parallel programs</u> The R&D strategy approach suggests the use of parallel and loosely related approaches to develop R&D programs. The decision to invest in a particular program need not be viewed as a "once-and-for-all" decision. Rather, the decision to work with some intensity on a program may be viewed as the exploration of a possibility aimed at finding out more about the chances of success and the costs and benefits of the particular program, if it were completed.

3.2.4.5 Normative theory   Marschak et al. [33] have formally extended these ideas to the pursuit of approaches with different intensities, isolating the peculiar way such different approaches relate to each other; to the case of many review points and multicomponent parallel developments; and to the case of many-valued preferences. As Hirschman argues quite convincingly, development projects are essentially "voyages of technological and administrative discovery" and as such, require an R&D approach such as the normative theory developed by Marschak et al. [33].

A fairly complete normative theory makes it possible to test reasonable conjectures about what good development strategy and organization look like. That is to say, it would be possible to find a variety of assumptions that yield models simple enough to analyze and for which the conjectures could be demonstrated to be true if, in fact, they are true. This requires a precise statement of the tested conjectures. If under certain assumptions a tested conjecture is invalid (the development strategy it describes is not good), the developer would have to decide whether or not such assumptions are met before he could accept the tested conjecture in formulating his own development strategy. Among the conjectures to be tested in this manner are that the more one learns in each approach to a development task, the more approaches it pays to pursue; and that successive increments in the scale of a development project yield smaller and smaller improvements in the project's payoff, suitably defined. In addition, the theory is a step in finding out what the production function of the knowledge-producing industry looks like. In particular, the theory would shed light on the critical question of returns to scale in the conduct of research and development, first with respect to individual projects--the most disaggregated research and development efforts--and ultimately by aggregation with respect to the national research and development effort. Furthermore, the theory helps in identification of various areas of ignorance about development in which intensive and critical inquiry would particularly pay off.

If certain properties of good development are particularly sensitive to assumptions about how knowledge is acquired, then it is particularly fruitful to make an empirical study of which assumptions are in fact satisfied. Such assumptions might have to do, for example, with the shape of the curve that relates the amount of money spent on development in a particular stage to the decrease in uncertainty (suitably measured) that this spending can bring about.

3.2.5 Logic trees

Conceptually, the decision tree approach can be used to solve R&D problems of the more complex type described above. However, as Raiffa points out, the decision tree in such a case is likely to become a "bushy mess" [34]. Unfortunately, some of the more powerful mathematical tools or solving multistage problems, such as linear and nonlinear programming, are severely limited in handling cases that involve uncertainty. The most viable approaches under uncertainty are probably dynamic programming and stochastic simulation.

3.2.6 Dynamic programming

Dynamic programming under uncertainty is a formal solution procedure which, at least for some decision problems, allowed the derivation of optimal decision strategies. Essentially, the procedure involves restating the complex multistage decision problem as a number of subproblems (one representing each stage within the planning horizon), each of which contains a few controllable or decision variables. The solution procedure is analogous to the "backward" solution employed in decision-tree analysis [35].

3.2.7 Simulation and Monte Carlo approaches

Unfortunately, generally applicable analytical solution procedures do not exist for certain classes of more complex problems. Thus, stochastic computer simulation is often regarded as the most realistic approach to tracing out the implications of alternative decision strategies. The simulation model can be as complex as a realistic statement of the decision problem. Interdependencies among actions and probability distributions can be specified in the model. The probability distribution of the overall performance criterion can be obtained for a given decision strategy by Monte Carlo sampling methods, that is, by sampling from the relevant probability distributions for each state of nature to obtain the value of the criterion function for a specific computer run. Successive runs with other samples allow approximation of the probability distribution of the particular strategy. Probability distributions for other strategies can be obtained in similar fashion. There is no guarantee that an "optimal" strategy can always be obtained with these methods, but "better" strategies can be derived [36].

3.2.8 Ranking and weighing

In some of the aforementioned techniques, a combination of several parameters has been used to describe the consequences of a given strategy. This approach has been shown inadequate in many cases since it does not reflect the preferences of the decision maker and hence it does not provide a guide to rational decisions. One approach presently employed to remedy the problems created by the reliance on a single attribute is to minimize or maximize the weighted sum of the expected values of all the relevant attributes. Besides the previously mentioned deficiencies of using an expected value of an attribute, this approach assumes only that the amounts of the individual attributes are important and the various ways they can be combined are unimportant. That is, evaluation is dependent only on the marginal probability distributions of the various attributes and not on their joint probability distribution. In many problems this assumption is unreasonable.

3.2.9 Target rate approach

Still another technique used by decision makers to determine which course of action to follow is the target rate. In this approach, acceptable levels of certain attributes are specified and, subject to attaining these levels, the expected value of another attribute is minimized or maximized. The acceptable level may have a specified probability associated with it. That is, for example, based on the decision maker's state of knowledge, only alternative courses of action which have a 0.99 probability of achieving the acceptance level are considered. This is similar to the value-impact approach.

3.2.10 Multiattribute utility approach (MAU)

3.2.10.1 Utility theory An R&D selection methodology may be based on the exploitation of utility theory and related decision analysis techniques. In some of the approaches mentioned earlier, a utility function has been used in place of the single attribute since the expected utility can be used as a guide to rational decision. The utility theory provides a mathematical model in which an attempt is made to measure preferences towards multiple objectives by means of numerical "utility functions". The concept of utility has been borrowed in decision analysis to establish a scale of expected monetary values of lotteries or business ventures. Assessment of numerical utilities can be used for non-monetary as well as monetary attributes. Many techniques have been suggested and used to assess utility functions of a single attribute.

Obviously, applications of mathematical decision-making models have, in the past, tended to use unidimensional and easily measurable objective functions. However, almost all decisions involve multiple criteria which are often subjective in nature, eluding easy qualification. The essence of good decision-making in such circumstances lies in trading off one goal properly applied in such situations only if these trade-offs can be expressed in quantitative form. Therein lies the value of multiattribute utility (MAU) models which provide a practical tool for the development of multidimensional objective functions.

Multiattribute utility theory provides a formal basis for describing or prescribing choices between alternatives whose consequences are characterized by multiple values of relevant attributes. Thus, analytic work on such problems requires that one obtains an objective function involving multiple measures of effectiveness to indicate the degrees to which these objectives are met. Such an objective function specifies a preference ranking of consequences, and allows one to identify the trade-offs between various combinations of levels of the different attributes. In a risk-free environment, one should choose the alternative course of action that maximizes (or minimizes) the objective function.

However, most decision problems involve uncertainties--and these uncertainties need to be either considered formally or informally in analyzing the problem. If one chooses to do this formally, it is necessary to specify an objective function with special characteristics in order to make the analysis for solving the problem tractable. For this reason, it would be convenient to be able to use the expected value of the objective function as a guide to identify the best alternatives. This is appropriate, given that one accepts the axioms of utility theory specified by Von Neumann and Morgenstern. The objective function is then a utility function. This utility function not only provides one with the necessary information to rank consequences and identify trade-offs between attributes, but it also follows the aforementioned axioms that one should choose the alternative that maximizes the expected utility [37].

The utility concept is theoretically sound, and the mathematical details are not involved. However, as mentioned earlier, the difficulty comes when one tries to specify reasonable procedures for obtaining multiattributed utility functions. The general approach followed by many people has been to make assumptions about preferences and then derive the functional form(s) of the utility function satisfying these assumptions. If the assumptions are verified, the functional form can be used to simplify the requisite assessments needed to specify the utility function. Often these assumptions are so involved that it is unreasonable to expect a decision-maker to ascertain whether or not they might be appropriate for a specific problem.

3.2.10.2 Isopreference maps This limitation can be removed by proper exploitation of the general properties of the multidimensional utility function in assessing it. One of the techniques used in evaluating multidimensional preferences is to use indifference maps or iso-preference curves in absence of uncertainty to transform the multi-variable function into a function of one variable via the employment of specific substitution rates. The technique has been tested and was found to be limited to situations in which the substitution rates are independent of the levels of the attributes.

3.2.10.3 Lexicography A lexicographic ordering is another procedure for evaluating multidimensional consequences under certainty. With this type of ordering, that is, one attribute dominates preferences, and only when there is indifference involving comparisons of this attribute is the second attribute considered. Hausner [38] presented an axiomatic structure providing this type of ordering.

The lexicographic formulation seems unreasonable for most decision problems, since the substitution rate between attributes is implied to be zero. However, in cases where both attributes can take on only a few discrete values and there is no uncertainty involved in the problem, a lexicographic ordering may be useful. Thrall [39] and Mellon [40] each suggests specific military examples for application of such preferences.

3.2.10.4 Additive approach The classical and most common approach for evaluating multidimensional consequences under certainty is the additive representation. If a set of attributes $\{X_1, X_2, \ldots X_i \ldots X_n\}$ has consequences $(x_1, x_2, \ldots, x_n)$, then the additive formulation, a real value v is assigned to each consequence $(x_1, x_2, \ldots, x_n)$ by

$$v(x_1, x_2, \ldots, x_n) = v_1(x_1) + v_2(x_2) + \ldots + v_n(x_n),$$

where $v_i(x_i) > v_i(x_i')$ if and only if x_i is preferred to x_i'.

Debreu [41] proved for $n > 3$ that if the substitution rate between attributes X_i and X_j is independent of all other attributes for all i and j, there exists a set of functions v_i, $i = 1, 2, \ldots, n$, such that v is larger for more preferable consequences. Luce and Tukey [42] gave an alternate proof of this proposition for $n = 2$, and Krantz [43] extended their work for arbitrary n. Miller [44] developed a similar assessment procedure and applied his work in choosing among various employment opportunities and in computer acquisition.

One major shortcoming of the additive approach is that the function v is not necessarily a utility function in the probabilistic sense. That is, the expected value of v cannot be used as an appropriate guide to decision making under uncertainty. Raiffa [34] suggests one approach to obtain a probabilistic index from the v function. This additive representation has been suggested by Hammond in the choice of a college and Aumann and Kruskal [45] for solving the assignment problem.

3.2.10.5 Additive utility approach In problems involving uncertainty, the usual approach for evaluating multidimensional consequences is the additive utility function. In n dimensions, this function may be written as

$$u(x_1,x_2,\ldots,x_n) = u_1(x_1) + u_2(x_2) + \ldots + u_n(x_n) \ ,$$

where $u_i(x_i)$ is a utility function defined over the X_i attribute.

For whole product sets, $X_1 \times X_2 \times \ldots \times X_n$ Fishburn [46] has proven that $u(x_1,x_2,\ldots,x_n)$ may be evaluated by an additive utility function if the desirability of any lottery, which is represented by $(\bar{x}_1,\bar{x}_2,\ldots,\bar{x}_n)$ where "-" indicates a random variable, depends only on the marginal probability distributions of the X_i's and not on their joint probability distribution. He has extended this idea using the same premise to include denumerable product sets, incomplete product sets, and interdependencies among some attributes. Pollak also derives necessary and sufficient conditions for additive utility functions [47].

In all these cases, the resulting utility function differs from the v function discussed above in that it is valid in the probabilistic sense; that is, expected utility should be used as a guide to rational behavior.

The main advantage to the additive utility function is its relative simplicity. The assessment of the n-dimensional utility functions, and--as previously mentioned--adequate systematic procedures do exist for assessing one-dimensional utility functions. A major shortcoming of this approach is the restrictiveness of the necessary assumptions. One would often expect the utility of a lottery to be dependent not only on the marginal distributions of the respective attributes, but also on their joint probability distribution. It is also difficult to determine whether or not the requisite assumptions would be reasonable in a specific real-world problem. This difficulty arises because the assumptions are stated in terms of the decision maker's preferences for probability distributions over consequences, rather than directly in terms of his preferences for consequences. Assumptions of the latter type have more operational significance. Often, the multidimensional utility function is assumed to be additive for convenience, rather than justified by testing the validity of the requisite assumptions. Many potential applications of additive utility functions, including cost-effectiveness problems, are suggested by Fishburn [48].

3.2.10.6 <u>The decision-maker preference approach</u> An alternative approach to assessing multidimensional utility functions involves assumptions about the decision maker's qualitative preferences for consequences. Based on these assumptions, it was shown by Keeney [6] how the two-dimensional utility function in certain cases could be evaluated from

$$u(x,y) = u_X(x) + u_Y(y) + k\, u_X(x)u_Y(y) \quad ,$$

where k was an empirically evaluated constant. This method has been further developed for n-dimensional utility functions.

Sufficient conditions have to be met in order to determine whether a multi-attribute utility function is of the above form. The number of conditions required increases linearly with the number of attributes. None of the conditions, however, requires the decision-maker to consider trade-offs between more than two attributes simultaneously or to consider lotteries over more than one attribute. In order to determine the single utilities for each attribute, several schemes have been developed for individuals and for a group of decision-makers.

In summary, the MAU R&D selection methodology involves the following steps:

(1) Consider the nuclear research center program and identify all possible measures of effectiveness or attributes, especially those pertinent to the program goals;

(2) Rank attributes and evaluate scaling factors and the multiplicative weight;

(3) Synthesize utility curves or preference patterns of all attributes by decision-makers and/or expert participation and then provide analytical expression for the utility functions;

(4) Check utility and preferential independence;

(5) Identify alternative programs or proposals (options) in a specific study area; and

(6) Maximize the MAU function to identify proposals.

3.2.10.7 <u>Merits of the MAU approach</u> From the review of the existing selection methodologies, it is apparent that existing methods suffer from many deficiencies, especially in dealing with practical selection of R&D programs of many attributes. Where intangibles tend to affect selection decisions, shadow pricing and social discount rates are only superficially used but are subject to controversy. Among the points which are in favor of an MAU approach are:

(1) Consideration of tangibles, monetized factors and intangibles;

(2) Ability to reduce multidimensional factors into one function useful for ranking order;

(3) Accommodation for decision-makers (program

managers, legislators, public groups) preferences;

(4) Amenability to analytic solutions and computer programming;

(5) Flexibility in iterative applications and in applying to all study areas of the nuclear research center program;

(6) Applicability to various problems similar in nature to the R&D selection program considered here;

(7) Simplicity of input information and directness of the output results.

3.2.11 Simple ranking and scoring approaches

There are a variety of simple approaches which are based on pure subjective judgment. The simplest is providing simple scores from 0 to 10, say for each program/proposal. A ranking of programs and proposal using various criteria of evaluation may be used with weights being assigned to each criterion. The weighted averages of each option can be then used for selection.

Another approach is the Q-sort method which can involve group decision-makers. The programs/proposals are ordered in groups of low, medium or high preference. The process is repeated several times after rounds of open discussions.

Pair comparison of evaluation criteria may be used to assign relative ranks and weights to criteria of evaluation and to measures of how a given program meets the criteria requirement.

3.2.12 Bayes decision approach

The traditional Bayesian decision approach [21] involves the use of the payoff matrices, logic diagrams and maximin and minimax techniques. An additional dimension is the use of value of information to assess the feasibility of postponing the decision until more precise information is obtained.

3.2.13 Verbal rating

To overcome the difficulty of selecting exact numbers for attribute levels or weights or the use of absolute numbers for ranking, one may assign a set of numbers each having a different level of possibility. Such fuzzy set accommodates for imprecision in values of attributes or utilities. The use of the fuzzy set theory in ranking, weighing or evaluation of utility or probability can add new information to the preferability among options. The relative degree of preference can be evaluated and the range of values in which a given option is preferred over others can be determined.

3.2.14 Decision-making in a fuzzy environment

Fuzzy sets theory is used to solve multiple attribute decision problems under uncertainty. A set of alternatives can be characterized by a number of attributes. By assigning verbal rating and weights to each of these attributes, one can compute weighted final ratings for each alternative which can be used to rank the alternatives. Since there is uncertainty involved with the ratings and weights, the verbal evaluation can be represented by variables in the form of fuzzy quantities which can be characterized by appropriate membership functions [49, 50].

Informally, a fuzzy set is a class of objects in which there is no sharp boundary between those objects that belong to the class and those that do not. For example, if $X = \{x\}$ denotes a collection of objects/points denoted generically by x, then a fuzzy set A in X ($A \subset X$) is a set of ordered pairs; that is,

$$A = \{(x, \mu_A(x))\}, \ x \in X \qquad (1)$$

where $\mu_A(x)$ is termed the grade of membership of x in A and $\mu_A: X \rightarrow M$ is a function transforming X to a space M called the membership space. We shall assume that M is the interval 0,1, with 0 for the lowest and 1 for the highest grades of membership. The symbol $\in$ is a shorthand for "belongs to" and $\subset$ is for "a subset."

If X = 0,1,2,... is a space comprising a collection of non-negative integers, then in this space the fuzzy set A of "several objects" may be defined (subjectively) as the collection of ordered pairs.

$$A = \{(3,0.6),\ (4,0.8),\ (5,1.0),\ (6,1.0),\ (7,0.8),\ (8,0.6)\}\ . \tag{2}$$

In dealing with any decision situation, the decision-makers have a priori defined objective or goal which they strive to achieve in an optimal way by taking the proper decision. Often the decision is limited by constraints such as budget constraints, regulations, or the law of the land.

The general properties of decision-making in a fuzzy environment have been studied by Bellman and Zadeh [51] who have defined fuzzy goals G for the decision, and fuzzy constraints C on the decision as fuzzy sets, with membership functions of $\mu_G(x)$ and $\mu_C(x)$, respectively. In this case, a fuzzy decision D can be viewed as an intersection of given goals and constraints. Consequently, $X = \{x\}$ is a set of possible alternatives, then the fuzzy goal or simply a goal, G, is a fuzzy subset of X, that is, $G \subset X$ and a fuzzy constraint or simply a constraint, C, is a fuzzy subset of X, that is, $C \subset X$. A decision is basically a choice or a set of choices selected from the available alternatives to achieve given goals subject to certain constraints. Then a fuzzy decision, D, can be defined as the set of alternatives resulting from the intersection of the goals and constraints; or D is a fuzzy subset of X, which is formed from a combination of a goal, G, and constraints, C,

$$D = G \cap C$$

where $\cap$ refers to intersection between G and C or literally the common members of G and C. The membership function, μ_D of D, is defined by

$$\mu_D(x) = \mu_G(x) \wedge \mu_C(x) \qquad (3)$$

where min $(\mu_G(x), \mu_C(x))$, $\qquad x \in X$.

The symbol $\wedge$ is referred to as the intersection, or the product minimum, of two functions.

In general, if there are n goals, $G_1, \ldots, G_n$ and m constraints, $C_1, \ldots, C_m$, the resultant decision is given by

$$D = (\bigcap_{i=1}^{n} G_i) \cap (\bigcap_{j=1}^{m} C_j) \qquad (4)$$

and similarly,

$$\mu_D = (\bigwedge_{i=1}^{n} \mu_{G_i}) \wedge (\bigwedge_{j=1}^{m} \mu_{C_j}) . \qquad (5)$$

The product of the membership functions may be used to get closer interdependence between goal and constraints [52]. Thus, a product-fuzzy decision is the fuzzy subset D_{pr} of X given by $D_{pr} = G \cap C$ with membership function $\mu_{D_{pr}}$ as

$$\mu_{D_{pr}}(x) = (\prod_{i=1}^{n} \mu_{G_i}(x)) \cdot (\prod_{j=1}^{m} \mu_{C_j}(x)) \quad . \tag{6}$$

An optimal decision x' is defined as any alternative in X which maximizes $\mu_D(x)$, that is

$$\mu_D(x') = \operatorname*{Sup}_{X} \mu_D(x) \tag{7}$$

where Sup is the supremum; that is any element with maximum membership function. Also, it is useful sometimes to assign weight to the constraints and goals in a decision-making process and hence one may define a convex-fuzzy decision as the fuzzy subset $\mu_{D_{co}}$ of X given by

$$\mu_{D_{co}} = \sum_{i=1}^{n} \alpha_i \mu_{G_i} + \sum_{j=1}^{m} \beta_j \mu_{C_i} \tag{8}$$

where $\{\alpha_i\}_{i=1}^{n}$, $\{\beta_j\}_{j=1}^{m}$, α_i, $\beta_j \geq 0$ and $\sum_{i=1}^{n} \alpha_i + \sum_{j=1}^{m} \beta_j = 1$.

Baas and Kwakermaak [53] proposed a special method for decision analysis of multiple-aspect alternative. In this method, $A_1, A_2, \ldots, A_m$ may denote the alternatives being compared and $a_1, a_2, \ldots, a_n$ as the different attributes (aspects) that these alternatives are to be judged upon. The relative merit of the aspect a_j for the alternative A_i is given by the fuzzy rating r_{ij} and the relative importance of each aspect is assessed by a fuzzy weighting coefficient, w_j of aspect a_j. In such a situation, the ranking of alternatives may be achieved on the basis of their weighted rating.

In order to evaluate alternative A_i in a fuzzy environment represented by fuzzy ratings r_{ij} and fuzzy weight w_j, consider the function g, mapping R^{2n} into R defined as

$$g(z) = \frac{\sum_{j=1}^{n} w_j r_{ij}}{\sum_{j=1}^{n} w_j} \tag{9}$$

where $z = (w_1, w_2, \ldots, w_n, r_1, r_2, \ldots, r_n)$ mapped on the product space R^{2n} defines a membership function μ_{z_i}, given by

$$\mu_{z_i}(z) = \left[\bigwedge_{j=1}^{n} \mu_{\underline{w}_j}(w_j)\right] \wedge \left[\bigwedge_{k=1}^{n} \mu_{R_{ik}}(r_{ik})\right] \ . \tag{10}$$

It is assumed that fuzzy rating of aspect a_j of alternate A_1 is represented by a membership function $\mu_{R_{ij}}(r_{ij})$, where r_{ij} takes its values on the real line R. Similarly, the relative importance of aspect a_j will be a fuzzy variable as well, represented by the membership function $\mu_{W_j}(w_j)$, where w_j takes its values on the real line R. The symbol $\wedge$ denotes the operation of taking the minimum (intersection).

Through the mapping g: $R^{2n} \rightarrow R$, the fuzzy set Z = (R^{2n}, μ_{z_i}) induces a fuzzy set $R_i = (\bar{R}, \mu_{\bar{R}})$ with membership function

$$\mu_{\bar{R}_i}(\bar{r}) = \sup_{Z:\ g(r) = \bar{r}} \mu_{Z_i}(z), \ \bar{r} \in R \ . \tag{11}$$

In order to compare these final fuzzy ratings to see which alternative is preferred, we define a fuzzy set (I, μ_I), whose membership function μ_I may be determined as follows: First, the conditional fuzzy set $(I, \mu_{I/\bar{R}})$ is defined with a membership function

$$\mu_{I/\bar{R}}(i \mid \bar{r}_1, \bar{r}_2, \ldots \bar{r}_m) = \begin{cases} 1 \text{ if } \bar{r}_1 \geq \bar{r}_j, \ \forall j \in I \\ 0 \qquad \text{otherwise} \end{cases} \tag{12}$$

where $\forall$ indicates all of j in the I mapped on R^m. The fuzzy final ratings define a fuzzy set $\bar{R} = (R^m, \mu_{\bar{R}})$ with a membership function

$$\mu_{\bar{R}}(\bar{r}_1, \bar{r}_2, \ldots \bar{r}_m) = \bigwedge_{i=1}^{m} \mu_{\bar{R}_i}(\bar{r}_i) \quad . \tag{13}$$

The fuzzy set $(R^m, \mu_{\bar{R}})$ and the conditional fuzzy set $(I, \mu_{I/\bar{R}})$ together induce a fuzzy set (I, μ_I) with membership function

$$\mu_I(i) = \operatorname*{Sup}_{\bar{r}_1, \bar{r}_2, \ldots, \bar{r}_m} \mu_{I/\bar{R}}(i \mid \bar{r}_1, \bar{r}_2, \ldots, \bar{r}_m) \wedge \mu_{\bar{R}}(\bar{r}_1, \bar{r}_2) \tag{14}$$

By this method one can get the order of preference of the alternative where the preferred alternative will have the degree of membership of unity. This order of the membership function μ_I decides which alternative is the best. However, the order gives only partial information about how much better the best alternative is compared to other alternatives, especially when there is more than one alternative with membership of one in the fuzzy set (I, μ_I). To overcome this problem, the mapping $h_i: R^m \; R$ is defined. The mapping h_i induces a fuzzy set $P_i = (R, \mu_{P_i})$ with membership function

$$\mu_{P_i}(p) = \sup_{\bar{r}_1, \bar{r}_2, \ldots \bar{r}_m :\, h_i(\bar{r}_1, \bar{r}_2, \ldots \bar{r}_m) = p} \mu_{\bar{R}}(\bar{r}_1, \bar{r}_2, \ldots \bar{r}_m), \; p \in R \,. \tag{15}$$

The membership function $\mu_{P_i}(p)$ may be used to judge the preferability of alternative A_i over the other alternatives.

In order to evaluate a membership function such as

$$\mu_Y(y) = \sup_{x \in R^n : f(x) = y} \bigwedge_{i=1}^{n} \mu_i(x_i), \; Y \in R \tag{16}$$

where $\mu_i(x_i)$, $i = 1,2,\ldots,n$ are given membership functions and $f(x)$ is a function of mapping R^n into R and $x = (x_1, x_2, \ldots, x_n)$ one needs to apply the following theorem. Assume that the membership functions μ_i, $i = 1,2,\ldots,n$ are piecewise continuously differentiable functions mapping R^n into R, each bounded non-negative and with finite support. Also, f is a continuously differential mapping of R^n into R. Now, suppose that the point $\hat{x} = (\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n) \quad R^n$ satisfies the following conditions:

(1) There exist derivatives of the following function

$$\mu'_i(\hat{x}_i) = \frac{d\, \mu_i(\hat{x}_i)}{dx_i} \neq 0 \tag{17}$$

and

$$f'_i(\hat{x}) = \frac{\partial f(x_1, x_2, \ldots, x_n)}{\partial x_i} \neq 0 \qquad (18)$$

(2) $$\mu_1(\hat{x}_1) = \mu_2(\hat{x}_2) = \ldots = \mu_n(\hat{x}_n) \ . \qquad (19)$$

(3) $\mu'_i(\hat{x}_i)/f_i(\hat{x})$ has the same sign for each $i \in \{1,2,\ldots,n\}$.

Then $\hat{x}$ is a strict relative maximum point of the mathematical programming problem,

$$\max \bigwedge_{i=1}^{n} \mu_i(x_i), \text{ subject to } f(x) = \hat{y},$$

where $\hat{y} = f(\hat{x})$. In order to apply this theorem to evaluate

$$\bar{r} = \frac{\sum_{j=1}^{r} w_j r_j}{\sum_{j=1}^{n} w_j} \qquad (20)$$

the partial derivatives f_{w_j} of the function $(\bar{r})$ with respect to w_j and the partial derivatives f_{r_j} with respect to r_j. We have

$$f_{\bar{r}_j}(r,w) = \frac{w_j}{\sum_{i=1}^{n} w_i} \qquad (21)$$

$$f_{w_j}(r,w) = \frac{r_j - \bar{r}}{\sum_{i=1}^{n} w_i} \qquad (22)$$

where $j = 1,2,\ldots,n$ and r and w denote the vectors

$(r_1, r_2, \ldots, r_n)$ and $(w_1, w_2, \ldots, w_n)$ respectively.

Thus, the value of $\bar{r}$ for which $\mu_{\bar{R}}(\bar{r}) = \mu_0$ is obtained by finding the numbers $\hat{r}_1, \hat{r}_2, \ldots, \hat{r}_n$ and $\hat{w}_1, \hat{w}_2, \ldots, \hat{w}_n$ such that $\mu_{R_i}(\hat{r}_i) = \mu_{W_i}(\hat{w}_i) = \mu_0$ for $i = 1,2,\ldots,n$ and such that $\mu'_{R_i}(\hat{r}_i)$ and $\mu'_{W_i}(\hat{w})/(\hat{r}_i - \bar{r})$ all have the same signs.

In order to compute the values of P_i for which

$$P_i = x_i - \frac{1}{m - 1} \sum_{\substack{j=1 \\ j \neq 1}}^{w} x_j \tag{23}$$

and

$$\mu_{P_i}(pi) = \mu_0$$

with μ_0 a given number we have to find numbers $\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_m$ such that $\mu_1(\hat{x}_1) = \mu_2(\hat{x}_m) = \ldots = \mu_m(\hat{x}_m) = \mu_0$ and $\mu_j(\hat{x}_j) =$ $j = 1,2,\ldots,m$ with $j \neq i$, all have the same signs, while $\mu'_i(\hat{x}_i)$ has the opposite sign.

4. MULTIATTRIBUTE UTILITY DECISION APPROACH

4.1. Introduction

In most complex multi-objective decision problems, more than one attribute or measure of effectiveness is needed to describe the possible consequences adequately. In problems of this type, it is not possible to maximize several objectives at once. It is not possible to cut costs and to increase benefits as well. So the decision maker is faced with a problem of trading off the achievement of one objective against another objective. The tradeoff issue becomes a preference value question and it requires the subjective judgment of the decision maker with reference to the real-world problems which involve risk and uncertainty. Multiattribute utility theory addresses this type of problem with a high degree of flexibility in its application. It considers systematically all available relevant information and utilizes explicitly the preferences of the decision maker. This is done by breaking the problem into parts, which are easier to analyze than the whole problem, and then putting the parts back together in a logical fashion [54]. For these reasons, the multiattribute utility theory has been chosen and adapted to suit the present decision problem.

In this chapter, historical background, the basic fundamentals of utility theory and the procedure for assessing utility functions are presented. The concept of multiattribute utility theory is also given focusing on a two attribute problem.

4.2. Historical Background

Bernoulli and Cramer, between 1700-1782, were the first to develop the essential elements of modern utility theory. Their hypothesis is that the unwillingness of individuals to accept bets even at actually better than fair odds reflected decreasing marginal utility of wealth. This idea of moral wealth as the measurement of a person's well-being is apart from any consideration of probability. Neither Bernoulli nor Cramer suggested a method for actually measuring utility functions, although Bernoulli suggested the maximization of expected utility as a valid principle [55].

Laplace, in 1825, defined probability as the ratio of favorable to possible outcomes, regardless of whether such a frequency ratio does or does not show a tendency to converge on a definite value and, in general, regardless of properties of the set of events for which such ratio was observed [56].

Ramsey, in 1928, was the first to express an operational theory of action based on the dual, intertwining notions of judgmental probability and utility. To Ramsey, probability is not the expression of a logical, rational or necessary degree of belief, but rather subjective degree of belief interpreted as operationally meaningful in terms of willingness to act or degree of overt betting behavior [57].

DeFinetti, in 1937, assessed a person's degree of belief by examining his overt betting behavior. By insisting on his assumption that series of bets are internally consistent or coherent, he demonstrated that a person's degree of belief must satisfy the usual laws of probability [58].

Von Neumann and Morgenstern, in 1947, developed the modern probabilistic theory of utility. This dealt exclusively with probabilities of canonical variety, that is, with drawings from a set where each element outcome is deemed equally likely [59].

Wald, in 1950, formulated the basic problem of statistics as a problem of action, and he analyzed the general problem in terms of normal-form analysis to select a best strategy for statistical experimentation and action when the true state of the world is unknown. He was primarily concerned with characterizing those strategies for experimentation and action which are admissible or efficient for wide classes of prototypical statistical problems. Wald's decision theory did not single out a best strategy but a family of admissible strategies [60].

Savage, in 1954, expressed the complete, purely probabilistic version of subjective doctrine. He suggested that intelligent individuals should in general attach to event weights which obey the laws of probability theory, regardless of how their degrees of belief are interpersonally controversial, and also regardless of which of these beliefs are stable in the mind of the decision maker [61].

Schlaifer, in 1967, adopted an approach that the decision maker structures his problem in terms of a decision flow diagram, assesses utilities and judgmental probabilities, and maximizes expected utility. He showed in detail how one can structure realistic problems and how one can elicit responsible judgmental inputs from the decision maker [62].

Raiffa, in 1970, suggested a scheme for the decision maker to use to organize and systematize his thinking when he encounters a situation in which he must make a decision. This approach describes how one who is faced with a problem of choice under uncertainty should go about choosing a course of action that is consistent with his personal basic judgments and preferences [34].

Fishburn, in 1972, proved that the utility of a multi-attribute alternative is equal to the sum of the utilities of individual attributes by the assessing of the traditional additive utility model. He assumed that various attributes are independent in their composite impact on total worth. He attempted to take the problem of attribute interaction into consideration through functional involving sums and weighted products of component utilities [38].

Keeney and Raiffa, in 1976, extended the utility theory to a higher level of applicability than ever before. They revealed the basic paradigm as a five step procedure consisting of (1) preanalysis where the problem, decision maker, and alternative are identified; (2) structural analysis where the problem anatomy is explicated as in a decision tree; (3) uncertainty analysis where probabilities are quantified; (4) utility analysis where a utility function is assessed; and (5) optimization analysis where expected utility is maximized to determine the best alternative. They examined preference over time and group utilities, respectively, with the purpose of demonstration of the adaptability of multiattribute utility concepts to these situations [37].

4.3. Fundamentals of Utility Theory

Assume that "A" is a decision to be made and a_i is one of the sets of option actions where the set is given as

$$A = \{a_1, \ldots, a_i, \ldots, a_m\} \qquad i = 1, \ldots, m \ . \quad (1)$$

Then, there are a set of attributes of concern

$$X = \{X_1, \ldots, X_j, \ldots, X_n\} \qquad j = 1, n \qquad (2)$$

and x_i represents a specific value of attribute X_i, where

$$x = \{x_1,\ldots,x_j,\ldots,x_n\} \quad j = 1,n\ . \tag{3}$$

Let us assume that x_1 is less preferred than x_2, which is less preferred than x_3, and so on, that is

$$x_1 < x_2 < x_3 < \ldots < x_n\ . \tag{4}$$

Now suppose that the decision maker is asked to state his preference between acts a' and a" where:

1. Act a' will result in consequence x_i with probability P_i' for i=1,n where $0 \le P_i' \le 1$, all i, and $\sum_i P_i'' = 1$.
2. Act a" will result in consequence x_i with probability P_i'' for i=1,n, where $0 \le P_i'' \le 1$, all i, and $\sum_i P_i'' = 1$.

Now, the following two options are presented:

I. Certainty option: receive x_i for sure.

II. Risky option: receive x_n (the best consequence) with probability Π_i or receive x_1 (the worse consequence with probability $1 - \Pi_i$).

The second option can be written as $\langle x_n, \Pi_i, x_1 \rangle$ and is called a lottery. Let $\Pi_n = 1$ and $\Pi_1 = 0$, and the Πs are

$$\Pi_1 < \Pi_2 < \ldots < \Pi_n\ . \tag{5}$$

Comparing equation 4 with 5, it can be seen that Π_i can be used as numerical scaling of x. If each x_i is associated with a scale Π_i, then the expected score for acts a' and a", labeled $\overline{\Pi}'$ and $\overline{\Pi}''$, respectively, are

$$\overline{\Pi}' = \sum_i P_i' \Pi_i \tag{6}$$

and

$$\overline{\Pi}'' = \sum_i P_i'' \Pi_i . \tag{7}$$

Since for act ', x_i occurs with probability P_i', and x_i is considered indifferent to a Π_i chance at x_n and complementary chance at x_i, then act a' is preferred to act a" if $\overline{\Pi}' > \overline{\Pi}''$ and is indifferent to a" if $\overline{\Pi}' = \overline{\Pi}''$.

Now, the Πs can be transformed into μs by means of a positive linear transformation.

$$\mu_i = a + b\,\Pi_i \qquad b > 0 \qquad i = 1,\ldots,n \tag{8}$$

then we have

$$\mu_1 < \mu_2 < \mu_3 < \cdots < \mu_n \tag{9}$$

and

$$\bar{\mu}' = \sum_i P_i' u_i = \sum P_i'(a + b\Pi_i) = a + b\overline{\Pi}' . \tag{10}$$

It should be noted that a positive linear transformation of Π does not change the ordering of the acts a' and a"; however, a nonlinear transformation of Π_i may no longer reflect the correct order of a' and a", but still is preserving the preference over x_i by $\mu_1 < \mu_2 < \cdots < \mu_n$.

One of the general characteristics of a utility function is monotonicity. That is

$$x_1 > x_2 \iff \mu(x_1) > \mu(x_2) \ . \tag{11}$$

This means that if preferences are monotone in x, then μ will be a monotone function of x.

Let a lottery yield consequences $x_1, x_2, \ldots, x_n$ with probabilities $P_1, P_2, \ldots, P_n$, respectively. So the expected value of a lottery is defined as

$$\bar{x} \equiv E(\tilde{x}) = \sum_{i=1}^{n} P_i x_i \tag{12}$$

where $\tilde{x}$ is the uncertain consequence and $\bar{x}$ is the expected consequence.

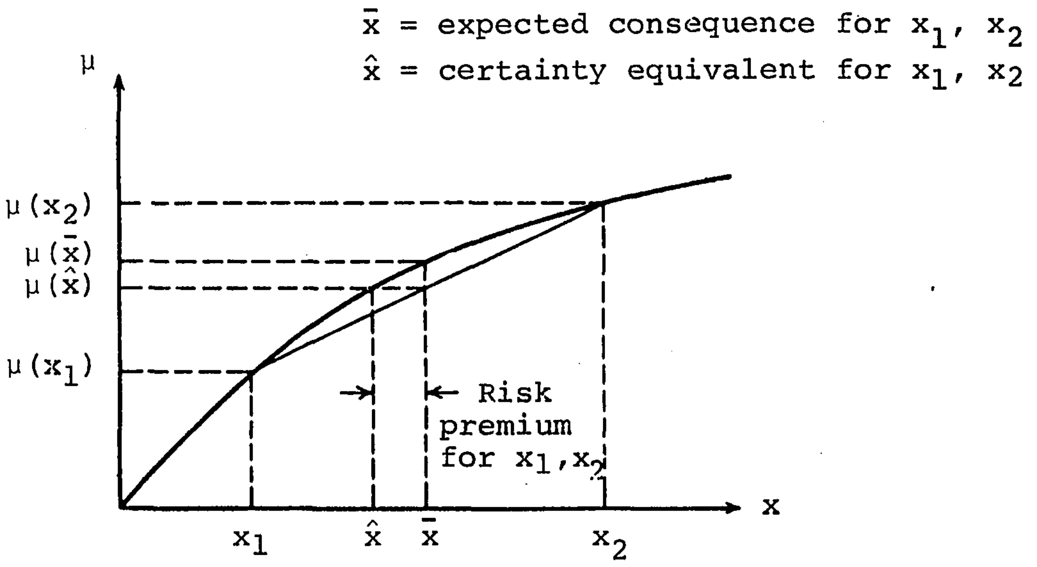


Figure 4.7. An increasing utility function exhibiting risk aversion

The expected utility of this lottery is defined as

$$E[\mu(\tilde{x})] = \sum_{i=1}^{n} P_i \ \mu(x_i) \ . \tag{13}$$

The expected utility is an appropriate index to rank order in choosing among lotteries.

A certainty equivalent of lottery L is an amount $\hat{x}$ such that between L and the amount $\hat{x}$ are indifferent for certain. Therefore, $\hat{x}$ is defined by

$$\mu(\hat{x}) = E[\mu(\tilde{x})] \tag{14}$$

or

$$\hat{x} = \mu^{-1} \ E[\mu(\tilde{x})] \ . \tag{15}$$

A decision maker is risk averse if he prefers the expected outcome of any lottery to the lottery itself. In such a case, the utility of the expected lottery must be greater than the expected utility of that lottery. This is represented as

$$\mu[E(\tilde{x})] > E[\mu(\tilde{x})] \tag{16}$$

or

$$\mu(\bar{x}) > \mu(\hat{x}) \ . \tag{17}$$

The decision maker is risk prone if

$$\mu[E(\tilde{x})] < E[\mu(\tilde{x})] \tag{18}$$

or

$$\mu(\bar{x}) < \mu(\hat{x}) \ . \tag{19}$$

The utility function of the decision maker is concave if he is risk averse, and it is convex if he is risk prone.

Two utility functions, μ_1 and μ_2, are strategically equivalent, written $\mu_1 \sim \mu_2$, if and only if they imply the same preference ranking for any two lotteries. It follows that if $\mu_1 \sim \mu_2$, there exist two constant a and b (a,b>0), such that

$$\mu_1(x) = a + b\mu_2(x) \quad \text{for all } x. \tag{20}$$

This property provides that changes in location or scale present no particular problem. Therefore, the utility function is usually scaled so that

$$\mu(x_1) = 1 \quad \text{and} \quad \mu(x_0) = 0 \tag{21}$$

where x_1, x_0 are the most and least preferred consequences, respectively.

A measure used to check for strategic equivalence of two utility functions is the risk aversion function

$$r(x) = -\frac{\mu''(x)}{\mu'(x)} = -\frac{d}{dx}[\ln \mu'(x)] \quad . \tag{22}$$

Two utility functions are strategically equivalent, if and only if they have the same risk aversion function. The decision maker's attitude toward risk can be determined by the risk aversion function such that

$$r(x) > 0 \longrightarrow \text{risk averse (concave)}$$

$$r(x) = 0 \longrightarrow \text{risk neutral (linear)}$$

$$r(x) < 0 \longrightarrow \text{risk prone (convex)}$$

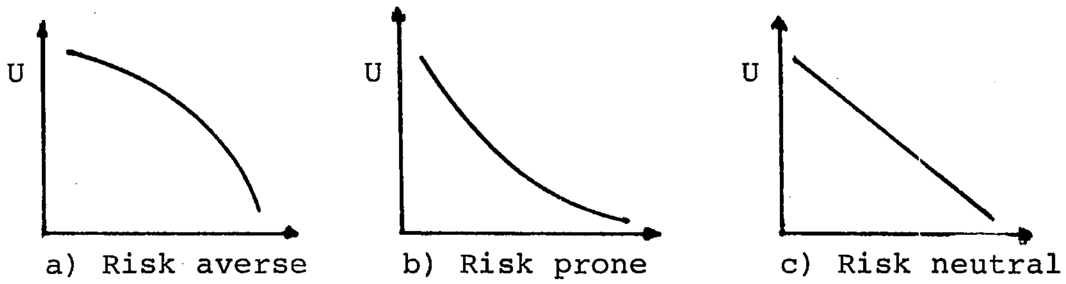


Figure 4.8. Risk properties of monotonically decreasing utility functions

Particularly useful utility functions displaying risk aversion (neutrality, proneness) are

$$\mu(x) = -e^{-cx}, \quad c > 0 \qquad \text{(averse)}$$

$$\mu(x) = x, \quad c = 0 \qquad \text{(neutral)}$$

$$\mu(x) = e^{-cx}, \quad c < 0 \qquad \text{(prone).}$$

4.4. A Procedure for Assessing Utility Functions

The basic ideas in assessing a utility function remain more or less the same for all procedures. That is, the specific points or objectives that must be considered and accomplished by any assessment procedure are essentially the same. The following should be done for assessing a utility function. The decision maker should reflect his preferences. Some values of x range over which preferences must be assessed as needed. The utility function μ should be determined whether or not is monotonic. It must be determined also whether μ is risk averse, risk neutral, or risk prone.

A splitting technique is useful to quantify the decision maker's subjective preferences and develop his utility function. Now x_1 is taken as the best consequence, and x_0 is taken as the worst consequence. We can set

$$\mu(x_1) = 1 \quad \text{and} \quad \mu(x_0) = 0$$

and we can define

$$\langle x_i, x_j \rangle = \langle x_i, 0.5, x_j \rangle \quad .$$

The decision maker is asked for a value of x such that he is indifferent between this value and $\langle x_1, x_0 \rangle$. This value is called $x_{.5}$ and

$$\mu(x_{.5}) = \tfrac{1}{2}\mu(x_1) + \tfrac{1}{2}\mu(x_0) = 0.5 \quad .$$

Next, we assess certainty equivalents for $\langle x_1, x_{.5} \rangle$ and $\langle x_{.5}, x_0 \rangle$, which we will designate as $x_{.75}$ and $x_{.25}$, respectively. And,

$$x_{.75} \sim \langle x_1, x_{.5}\rangle \text{ and } x_{.25} \sim \langle x_{.5}, x_0\rangle$$

$$\mu(x_{.75}) = \tfrac{1}{2}\mu(x_1) + \tfrac{1}{2}\mu(x_{.5}) = 0.75$$

$$\mu(x_{.25}) = \tfrac{1}{2}\mu(x_{.5}) + \tfrac{1}{2}\mu(x_0) = 0.25 \ .$$

We now have the points $(x_0,0)$, $(x_{.25},0.25)$, $(x_{.5},0.5)$, $(x_{.75},0.75)$, and $(x_1,1)$ which may be plotted and checked against the assumptions. As a consistency check, questions may be asked such as: is the decision maker indifferent between $x_{.5}$ and $\langle x_{.75}, x_{.25}\rangle$?

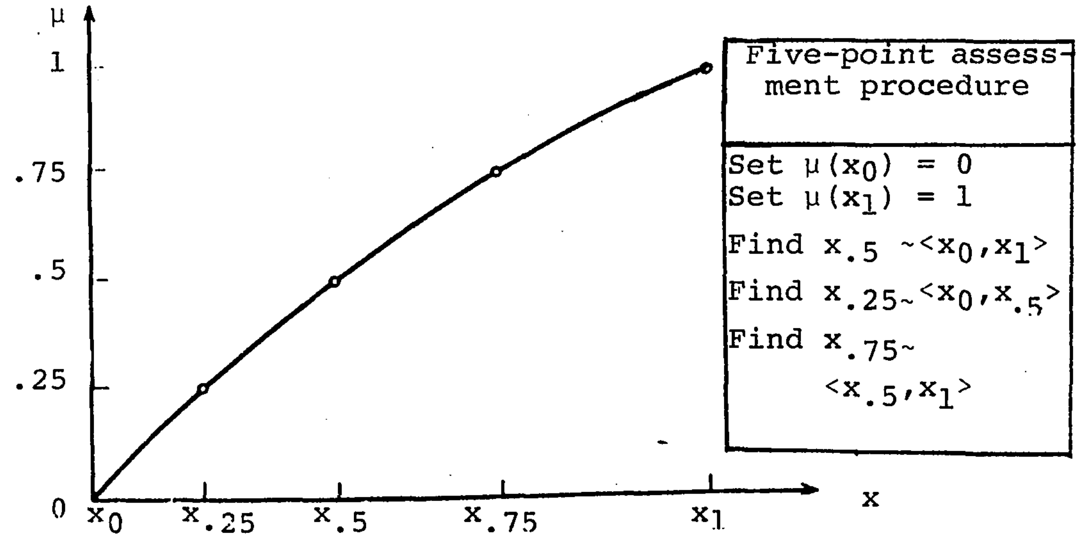


Figure 4.9. A five-point assessment procedure for utility functions

After checking for consistency, next we try to find a parametric family or utility function which satisfies the assumptions. Using the certainty equivalents (points), we try to determine a specific member of the family which represents the decision maker's preferences. If μ is not monotone in X, it will probably be necessary to find two utility functions μ_1 and μ_2 so that

$$\mu(x) = \begin{cases} \mu_1(x) & \text{for } x \leq x_m \\ \mu_2(x_2) & \text{for } x \geq x_m \end{cases}$$

where μ is increasing to x_m and then decreasing after x_m.

4.5. Multiattribute Utility Function

Many of the concepts of importance in multiattribute utility theory can be illustrated with two attribute problems. Therefore, to avoid unnecessary complication and detail, we focus on two attribute problems.

In the multiattribute development, the concept of utility independence is needed to express the utility function as a function of the individual conditional utility function such that

$$\mu(x_1, x_2, \ldots, x_n) = f[f_1(x_1),\ f_2(x_2), \ldots, f_n(x_n)]$$

where f_i is a function of attribute X_i only and where f is preferably in an additive or multiplicative form.

The attribute Y is said to be utility independent of the attribute Z if the conditional preferences for lotteries on Y given z do not depend on the particular level of z. This implies the functional form

$$\mu(y,z) = g(z) + h(z)\ \mu(y,z)$$

for all y and z, where g(.) and h(.)>0. It should be noticed that Z is not necessarily utility independent of Y. There is a single utility function structure over Y when Y is utility independent of Z. Finding this simplifies the assessment of $\mu(y,z)$. One simple form is the additive independent form

$$\mu(y,z) = k_Y\mu_Y(y) + k_Z\mu_Z(z)$$

where,

1. $\mu_Y(y)$ is conditional utility function on Y with

$$\mu_Y(y_0) = 0 \quad \text{and} \quad \mu_Y(y_1) = 1 \quad ;$$

2. $\mu_Z(z)$ is conditional utility function on Z with

$$\mu_Z(z_0) = 0 \quad \text{and} \quad \mu_Z(z_1) = 1 \ ;$$

3. $K_Y = \mu(y_1,z_0)$;
4. $K_Z = \mu(y_0,z_1)$;
5. $\mu(y_0,z_0) = 0, \mu(y_1,z_1) = 1$; and
   $\mu(y_1,z_0) > \mu(y_0,z_0),\ \mu(y_0,z_1) > \mu(y_0,z_0)$.

The additive utility function implies that Y and Z are mutual utility independent. However, the converse is not true.

Mutual utility independence does not imply that the utility function is additive, but it implies a multilinear representation such as

$$\mu(y,z) = \mu(y,z_0) + \mu(y_0,z) + k\mu(y,z_0)\ \mu(y_0,z)$$

or

$$\mu(y,z) = k_Y\mu_Y(y) + k_Z\mu_Z(z) + k_{YZ}\mu_Y(y)\ \mu_Z(z)$$

where

1. $\mu(y,z)$ is scaled so that $\mu(y_0,z_0) = 0$,
   $\mu(y_1,z_1) = 1$ and
   $\mu(y_1,z_0) > \mu(y_0,z_0)$, $\mu(y_0,z_1) > \mu(y_0,z_0)$;
2. $\mu_Y(y)$ is a conditional utility function on Y with $\mu_Y(y_0) = 0$ and $\mu_Y(y_1) = 1$;
3. $\mu_Z(z)$ is a conditional utility function on Z with $\mu_Z(z_0) = 0$ and $\mu_Z(z_1) = 1$;
4. $k_Y = \mu(y_1,z_0)$;
5. $k_Z = \mu(y_0,z_0)$;
6. $k_{YZ} = 1 - k_Y - k_Z$ and $k = k_{YZ}/k_Yk_Z$.

The multilinear form has a strategically equivalent multiplicative representation provided that $K \neq 0$ as

$$\begin{aligned}\mu'(y,z) &= k\mu(y,z) + 1 \\ &= k\mu(y_0,z_0) + k\mu(y,z_0) + k^2\mu(y_0,z)\ \mu(y,z_0) + 1 \\ &= [k\mu(y,z_0) + 1][k\mu(y_0,z) + 1] \\ &= \mu'(y,z_0)\ \mu'(y_0,z)\ .\end{aligned}$$

The utility function can be represented by an additive form such that

$$\mu(y,z) = \mu(y,z_0) + \mu(y_0,z)$$

if

1. $K = 0$;
2. Y and Z are mutually utility independent;
3. $<(y_3,z_3),\ (y_4,z_4)> \sim <(y_3,z_4),\ (y_4,z_3)>$
   for some y_3,y_4,z_3,z_4 such that (y_3,z_3) is not indifferent to either (y_3,z_4) or (y_4,z_3).

For the general case with more than two attributes, $X_1,X_2,\ldots,X_n$ are mutually utility independent if every subset of $\{X_1,X_2,\ldots,X_n\}$ is utility independent of its complement. If attributes $X_1,\ldots,X_n$ are mutually utility independent, then

$$\begin{aligned}\mu(x) = \sum_{i=1}^{n} k_i\mu_i(x_i) &+ K\sum_{i=1}^{n} k_ik_j\mu_i(x_i)\mu_j(x_j)\\ &+ K^2\sum k_ik_jk_e\mu_i(x_i)\mu_j(x_j)\ \mu_e(x_e) + \ldots\\ &+ k^{n-1}k_1k_2\ \ldots\ k_n\mu_1(x_1)\ \mu_2(x_2)\ \ldots\\ &\quad \mu_n(x_n)\end{aligned}$$

where,

1. $\mu(x_1^0,x_2^0,\ldots,x_n^0) = 0$ and $\mu(x_1^1,x_2^1,\ldots,x_n^1) = 1$;
2. $\mu_i(x_i)$ is conditional utility function on X_i with $\mu_i(x_i) = 0$ and $\mu_i(x_i^1) = 1$ for $i = 1,2,\ldots,n$;

3. $k_i = \mu(x_i^1, \bar{x}_i^0)$, where $\bar{x}_i^0$ indicates $(x_1^0, x_2^0, \ldots, x_n^0)$;

4. K is a scaling constant that is a solution to

$$1 + K = \prod_{i=1}^{n} (1 + K\, k_i) \quad .$$

It should be noted that when

i) $\sum_{i=1}^{n} k_i = 1$, then $K = 0$, the utility is additive with

$$\mu(x) = \sum_{i=1}^{n} k_i \mu_i(x_i) \; ;$$

ii) $\sum_{i=1}^{n} k_i \neq 1$, then $K \neq 0$, the compact form

$$K\mu(x) + 1 = \prod_{i=1}^{n} [K\, k_i \mu_i(x_i) + 1] \quad .$$

4.6. Development of Selection Criteria

It is important in any decision problem to find the set of attributes which covers all important aspects of the problem and meets the overall objective. This implies that the attributes must be useful for this purpose. The attributes must be meaningful to the decision maker, so that he can understand the implications of the alternatives. The final set of attributes should not have any redundancies and the attributes should be defined to avoid double counting of consequences.

Ideally, the method should accurately reflect the decision maker's true feelings, helping him to better understand what he really wants. This proves most useful when the number of objectives confronting the decision maker is so large that the trade-off synthesis is too difficult to accomplish without computational aid.

Early in the process of decision making, objectives must be identified and clearly defined. An objective should specify an area of concern to the decision maker, indicating the direction of improvement that he is seeking. If a specific level of achievement is desired, then this would properly be called a goal instead of an objective. What is really wanted here is a direction of desired improvement rather than some specific goal.

Broad objectives should be divided into sub-objectives for which attributes may easily be defined. An attribute is a measurement that refers to what degree an objective is met. An attribute may be either scalar or vector valued. Any attribute which does not directly measure an objective is called a proxy attribute.

Keeney and Raiffa suggest that a set of attributes should have the following desirable properties [37]:

(1) Completeness: All important aspects of the problem should be covered. If the decision maker knew the value of every attribute, he should be able to tell how well the overall objective is met.

(2) Operational: The attributes should be such that they can be meaningfully used in the analysis.

This implies that the attributes should be meaningful to the decision maker and should aid in explanations to others.

(3) Decomposable: To make the problem tractable, it is usually necessary to break the utility assessments into smaller parts, each part containing only a few attributes. One needs to be able to group some attributes together in order to reduce dimensionality of the problem.

(4) Non-redundancy: Attributes should be defined so that different attributes aren't measuring the same thing. One is when one attribute measures part of what another attribute measures. The other is when some attributes deal with input and some with output.

(5) Minimum size: The set of attributes should be kept as small as possible since the complexity of the problem increases vastly as the number of attributes increases.

Great care must be taken in choosing the attributes which will be considered. All of the subsequent analysis depends on this vital step. A poorly chosen set can easily make the problem intractable or lead to problems of inconsistency.

Once the attributes have been carefully chosen, one needs to evaluate the multiattribute utility function. The proposed method is to break the set of attributes into smaller sets which may be more easily assessed. To successfully break up the set of attributes, one needs the concepts of preferential independence and utility independence. Once the smaller sets are obtained, it is necessary to be able to assess a one-dimensional utility function. Hence, some definitions and results follow with the basic ideas of unidimensional utility theory.

The paradigm shown in the following figure illustrates the steps to be followed in the definition of the factors to be used in the SNRC program evaluation process. The general objectives of acquiring a research reactor will be identified in Chapter 5. A measure of efectiveness can be

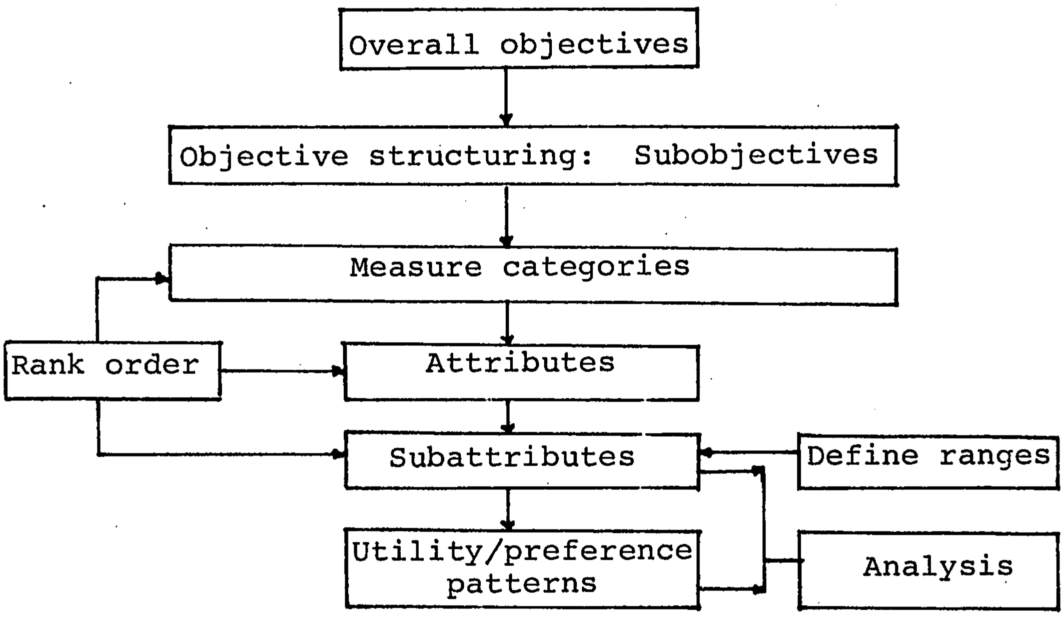


Figure 4.10. MAU paradigm

used as a metric of the level of achievement of each objective. The measure categories carry different importance and hence a tradeoff process may be used to rank order them. The process involves fixing each measure at its worst level and selecting which one would be raised to its best level on the expense of not raising the level of other measures. The selected measure is then eliminated and the process is repeated until a rank is assigned to all measures.

5. STRUCTURING THE PROBLEM

5.1. Introduction

A major step in a decision-making process is to define clearly the decision problem, especially when the problem is one of value tradeoffs. In essence, the decision maker is faced with a problem of trading off the achievement of one objective against that of another objective. Systematically structuring such tradeoffs is a very important process. This is done by definition of the overall objective, the options available and the constraints or environment within which the decision can be made.

5.2. Objective

The goal of the study is to assess means of enhancing the role of Saudi Arabia as an active participant in the peaceful utilization of nuclear energy. This can be narrowed down to a more realistic practical objective of establishing a nuclear research center which would provide adequate research facilities, a place where scientists and engineers can be trained, a center where interaction between Saudi Arabia scientists and the nuclear community from other countries can occur, and services to the local industrial community. Furthermore, the center may develop into a centralized information agency. Part of the strategy to achieve the goal is to build a research facility. Thus, the specific objective of the decision-making process considered here is the "Selection of an appropriate research reactor facility (RRF) for Saudi Arabia." This means selecting from among various available research reactor facilities to determine the facility which can best meet the need of the country.

The decision analysis in this case would require assessment of the following issues:

(1) types of reactors and auxiliary facilities (options);

(2) possible areas of research (basic and applied) to be conducted in the facility; and

(3) possible services that can be provided to Saudi Arabia.

5.3. Structuring the Objective

Structuring the decision problem means to generate an appropriate set of objectives, subobjectives, and measure of effectiveness to indicate the degree to which these objectives might be achieved by various alternatives. Clearly, the overall objective of the decision problem at hand is "to select the most appropriate research reactor facility (RRF) for Saudi Arabia." However, this objective is too broad to be operationally useful in analyzing the alternatives. It must be divided into a number of lower-level more specific objectives which can be easily assessed. An appropriate set of objectives can be drawn from the required facility characteristics; specifically, the RRF must be

(a) economical;

(b) technologically feasible;

(c) safe;

(d) capable of providing all or most anticipated functions and services for the country; and

(e) compatible with the local environment.

This set of objectives is oversimplified; nevertheless, it is still too general to be of practical use. Hence, subobjectives are developed and divided further to a level that allows the assignment of a set of measures and sub-measures of effectiveness to each subobjective. The goal, objectives, and subobjectives are shown in Figure 5.1. The corresponding measures and attributes are given in Figures 5.2 through 5.7 wherein further branching is made to assume completeness of the assessment.

5.4. Definitions of Categories, Attributes and Subattributes

Specific definitions are developed for each variable to assure the uniqueness of each measure/attribute/subattribute. This is necessary to avoid confusion in the interpretation of the effectiveness of each variable, to eliminate double counting or overlap of attributes and to aid identifying the effects of variation in the level of each attribute.

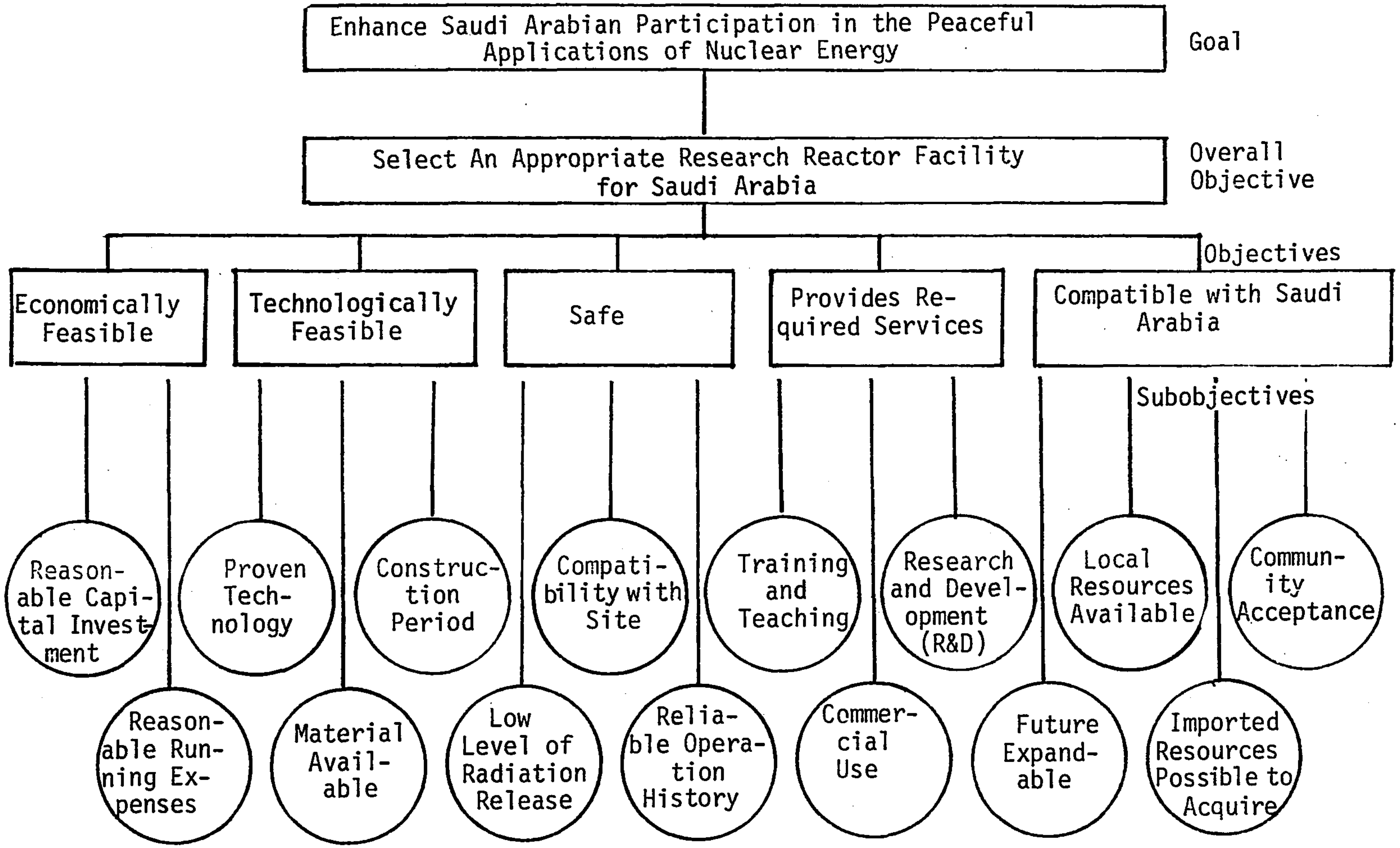


Figure 5.1. Structuring the objectives

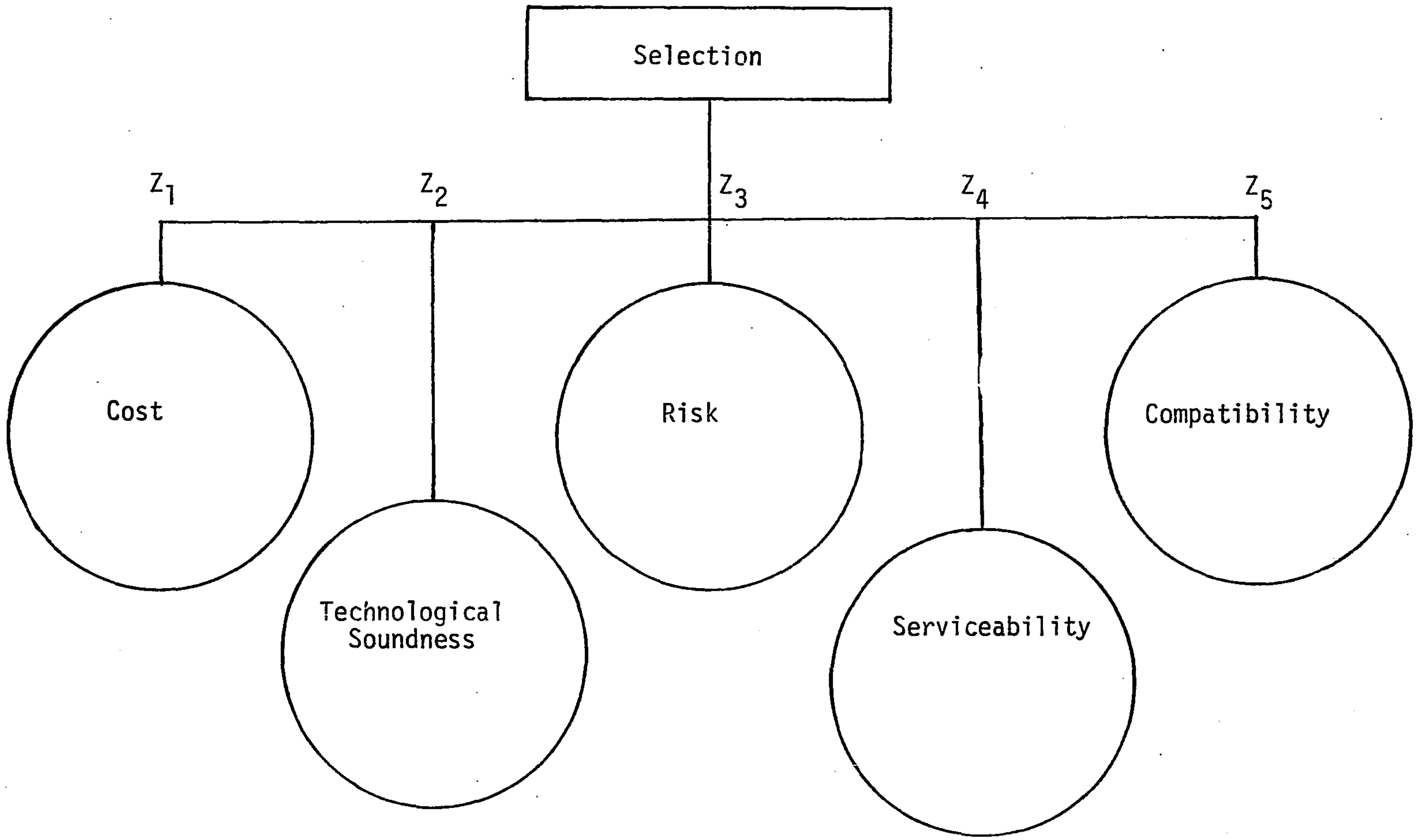


Figure 5.2. Measures of effectiveness of the selection process

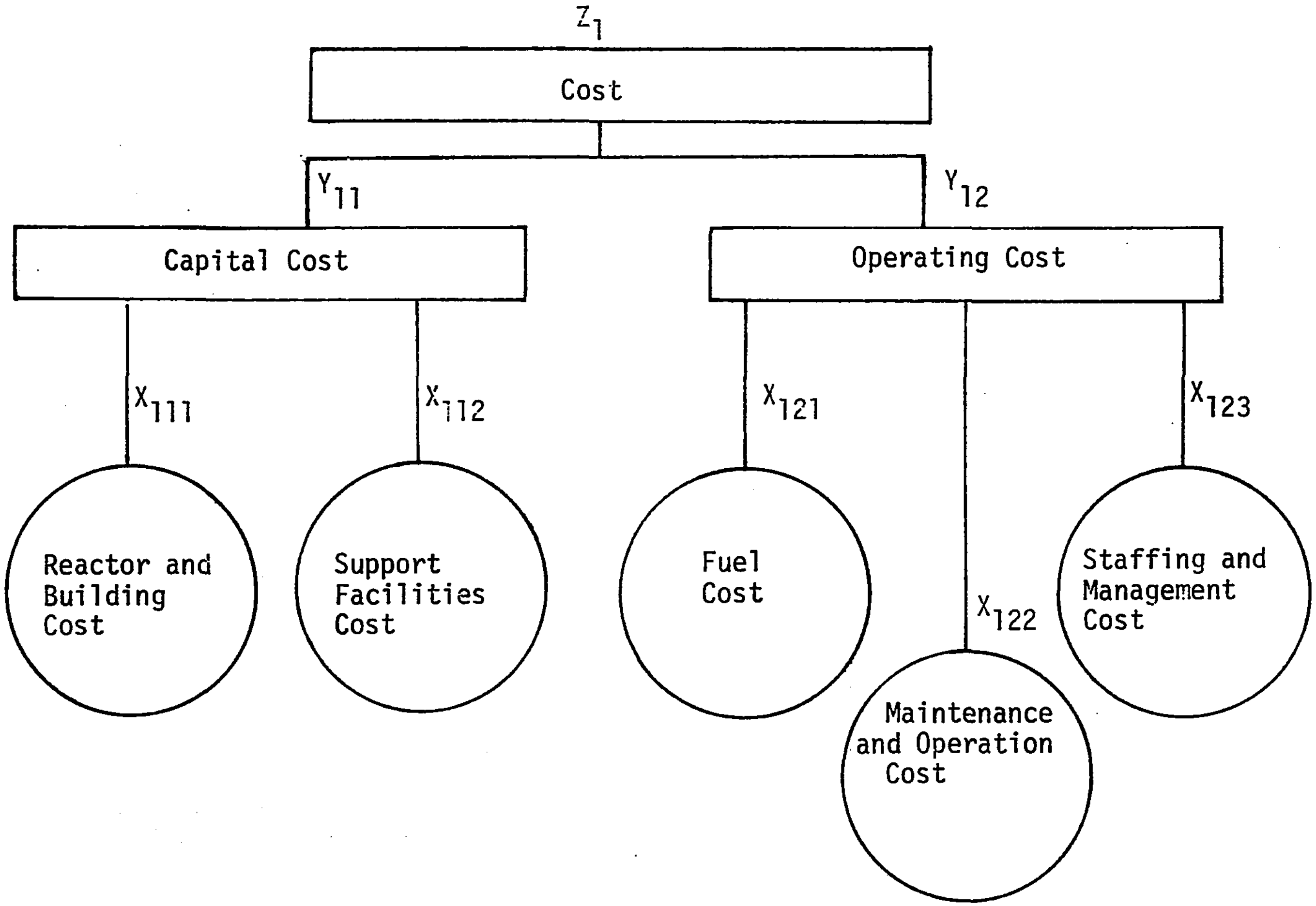


Figure 5.3. Cost attributes and subattributes

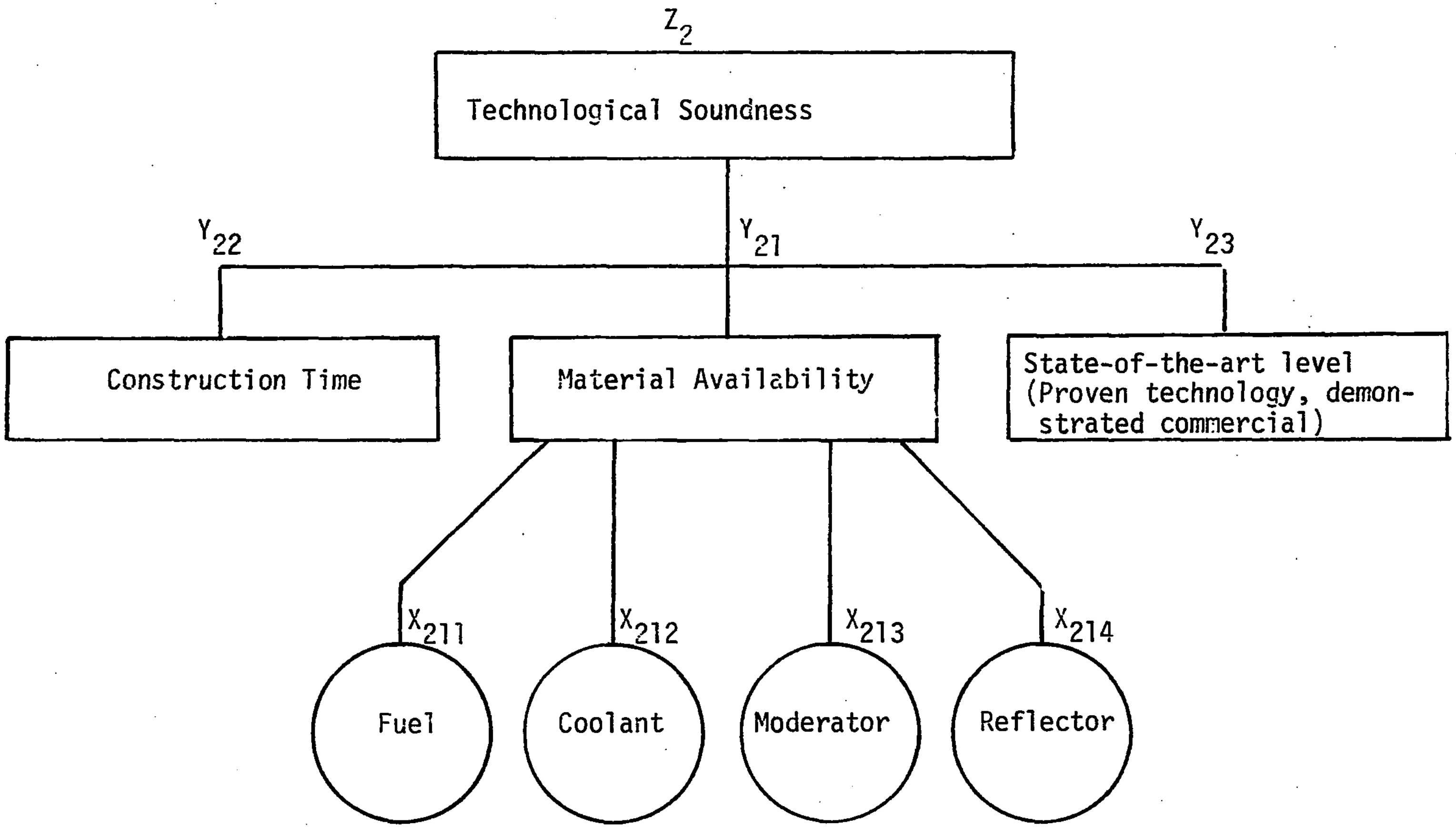


Figure 5.4. Attributes and subattributes associated with technological soundness

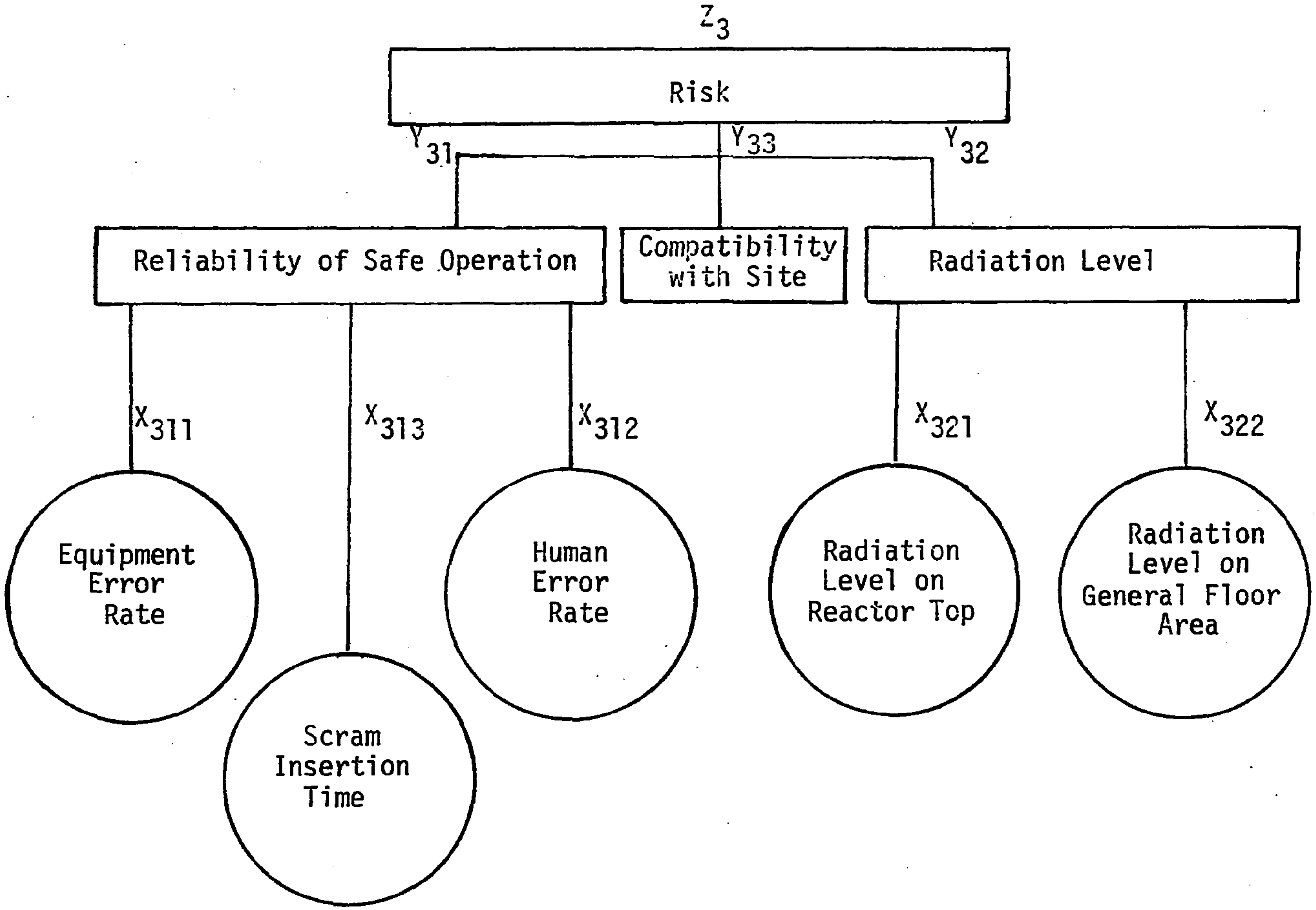


Figure 5.5. Attributes and subattributes associated with risk

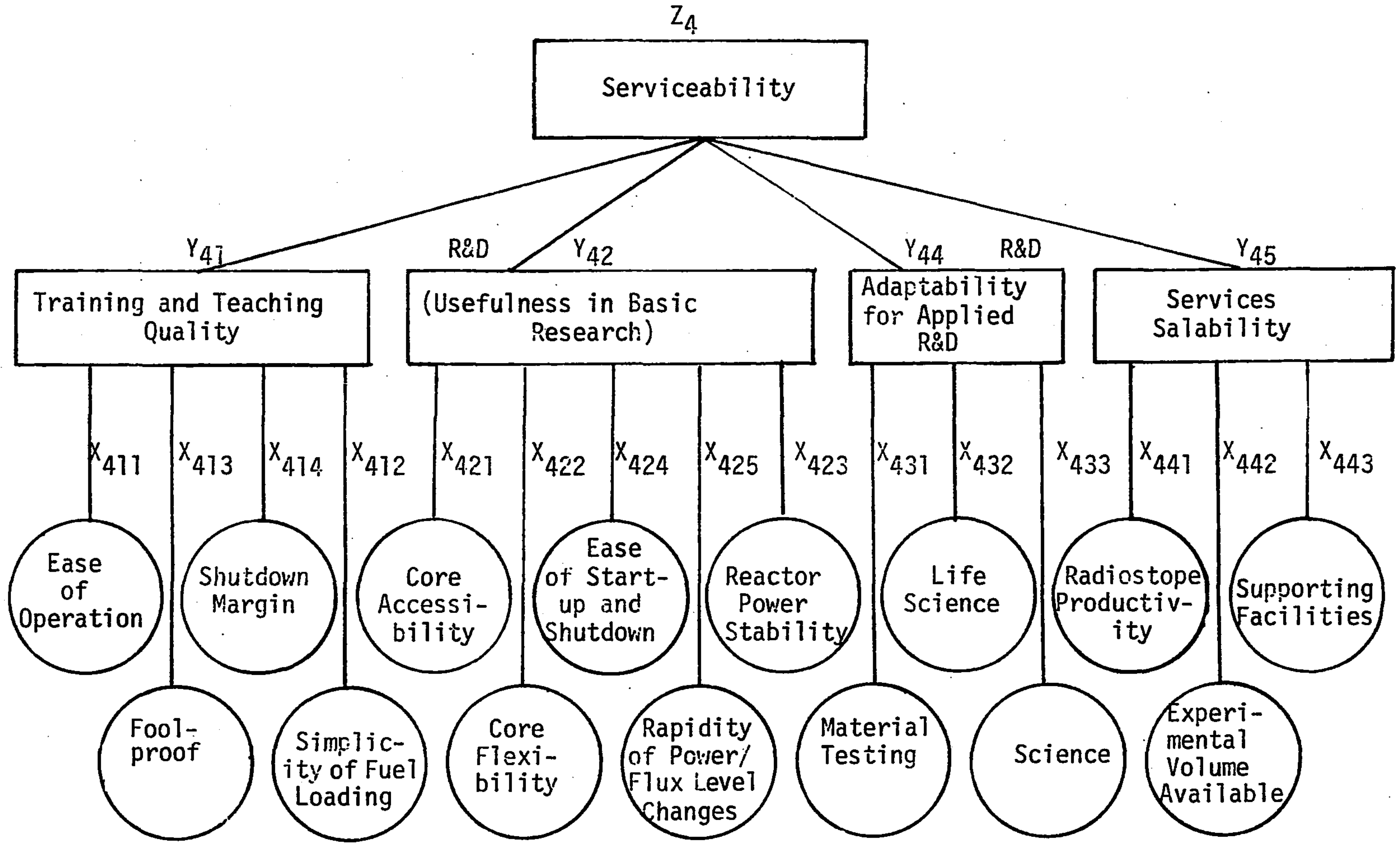


Figure 5.6. Attributes and subattributes of serviceability

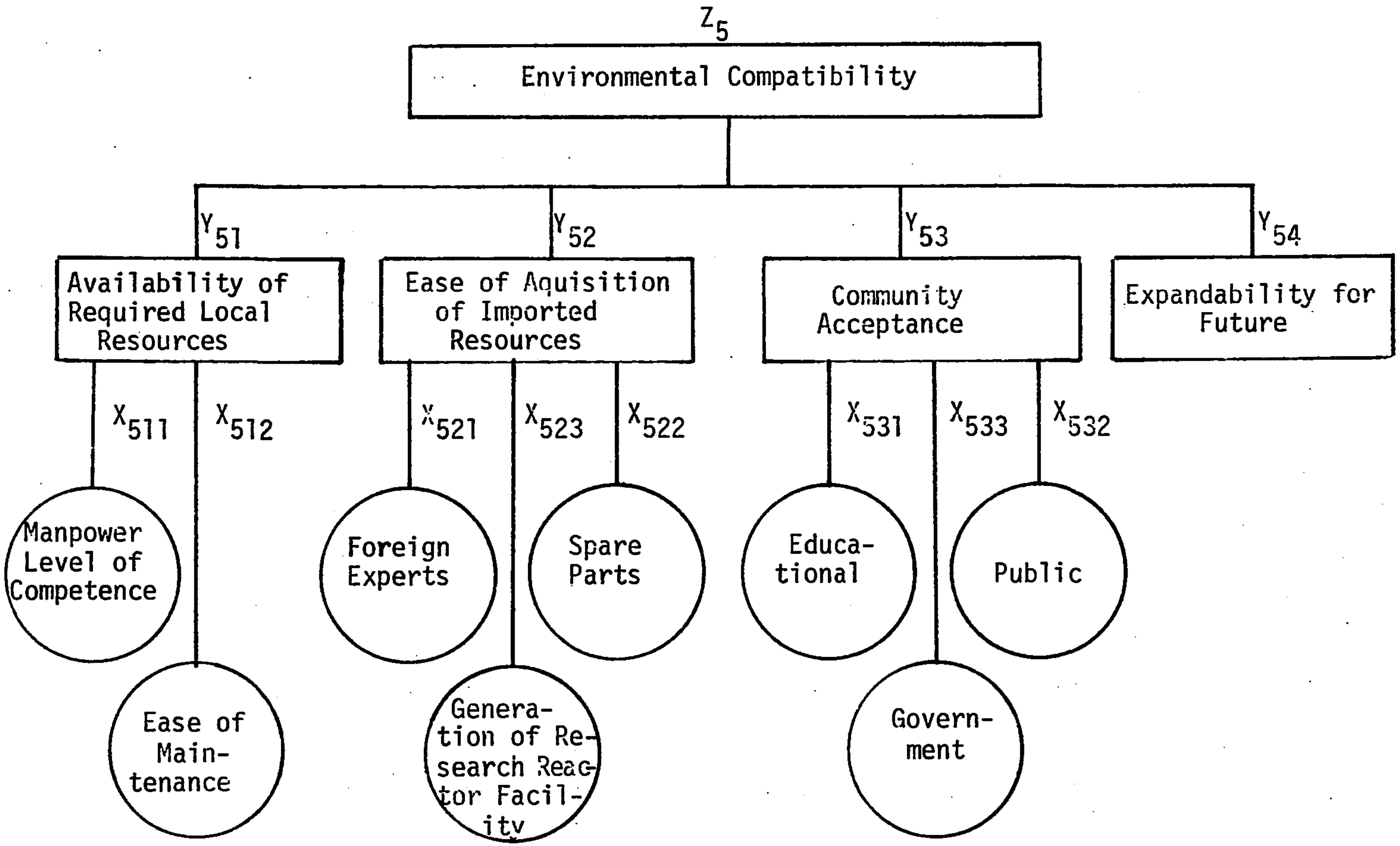


Figure 5.7. Attributes and subattributes of environmental compatibility

5.4.1. Definition of cost category, attributes and subattributes

Category (Z_1) provides a measure to evaluate the comparative cost of various alternative research reactor facilities in pure economical terms. The category consists basically of two major attributes: capital cost and operating cost. Each of these broad attributes is in turn based on a number of subattributes; namely,

Y_{11} - Capital cost. It means the total cost for building a research reactor facility which does not only include land, design and construction costs, but also associated costs of equipment and supporting facilities.

X_{111} - Reactor and building cost. This includes costs of land, land preparation, reactor hazard evaluation, design and construction work including core, core container (pool or tank), beam tubes, thermal column, shielding containment, offices, heat removal, etc.

X_{112} - Supporting facilities cost. This includes hot cells, spectrometers, multichannel analyzers, neutron sources, radioisotope laboratories, counting rooms, repair shop, and other handling equipment necessary to obtain information regarding the effects of radiation.

Y_{12} - Operating cost. This is a yearly budget to sustain the operation of the research reactor center. It includes fuel cost, operating and maintenance expenses, and staffing and management salaries and overhead.

X_{121} - Fuel cost. It includes the cost of fabrication, reprocessing, fuel consumption, enrichment, and transportation (universities in U.S.A. usually do not pay the inventory and burn up costs).

X_{122} - Maintenance and operation cost. This is the cost of consumable equipments, supplies required for the reactor and supporting facilities, utilities, and other direct (does not include personnel salaries and overhead) costs for maintaining and operating the center.

X_{123} - Staffing and management cost. It includes administrative, operating, safety and health physics staff. This cost varies with the type of reactor and operation time.

5.4.2 Definition of technological soundness category, its attributes, and subattributes

The evaluation of technological soundness, Z_2, of various research reactor systems can be characterized by the following attributes: (1) material availability; (2) construction time; and (3) the state-of-the-art of commercially proven systems. These attributes and subattributes provide adequate confidence that the selected system will be most desirable from a technological point of view.

Y_{21} - Material availability. The material requirements for a research reactor are highly dependent on the availability of material sources and the continuity of supply. This includes the availability of fuel, coolant, moderator, and reflector.

X_{211} - Availability of fuel. This is heavily based on the degree of fuel enrichment.

X_{212} - Availability of coolant. This is determined by the type of coolant.

X_{213} - Availability of moderator. Availability is determined by the type of moderator.

X_{214} - Availability of reflector. Availability is determined by the type of reflector.

Y_{22} - Construction time. The time period needed from the start of construction to the first fuel loading.

Y_{23} - State-of-the-art level. The scope of this attribute is to provide a measure of the commercial viability of the reactor considered for evaluation based upon degree of proven technology employed in the design and the extent of operation experience with such a system.

5.4.3. Definition of risk category, attributes and subattributes

The scope of this category (Z_3) is to evaluate research reactor systems from a safety viewpoint. It takes into consideration the following attributes

(1) reliability of operation;

(2) radiation level; and

(3) compatibility of a particular reactor with the selected site.

These attributes are divided into subattributes that provide a measure of effectiveness that delineates distinction between the alternatives.

Y_{31} - Reliability of operation. This depends on equipment and human performance as found from operation experience of each alternative facility, and depends on scram insertion time in case of an accident.

X_{311} - Equipment failure rate. Measures rate of failure of reactor system equipment which is important to the prevention of accidents.

X_{312} - Human error rate. This is a measure of failure of the operational personnel to perform as required to keep the reactor operation safe.

X_{313} - Scram insertion time. This is the time period required for the control rod banks to move to shut the reactor quickly in the event of emergency to assure safe shutdown of the reactor.

Y_{32} - Radiation level. This provides a measure of the radiation level on the reactor top and on the general floor area inside the reactor building to assure safe operation for staff, researchers, and public.

X_{321} - Radiation level on reactor top.

X_{322} - Radiation level on general floor area.

Y_{33} - Compatibility with selected site. The amount of flexibility in locating a particular type of research reactors taking into consideration the environment surrounding the area where the center will be located.

5.4.4 Definition of serviceability category, attributes and subattributes

Serviceability (Z_4) provides a measure of how a research reactor can best provide the required services and provide for the varied needs of the country. The evaluation measures of alternatives in this case include the following attributes:

(1) training and teaching quality;

(2) usefulness in basic research;

(3) adaptability for applied research and development; and

(4) services salability.

Y_{41} - Training and teaching quality. This attribute comprises all factors that give adequate confidence that the reactor will be suitable for training and teaching programs. The subattributes include ease of operation, fool-proof, inherent safety, shutdown margin, and simplicity of fuel loading.

X_{411} - Ease of operation. This is a measure of the degree of ease for enabling inexperienced students and non-technical personnel to operate the reactor with minimum training.

X_{412} - Simplicity of fuel loading. Fueling, refueling, and fuel handling should be simple and easy. This will depend mainly on the design of the reactor and on the available tools and procedures.

X_{413} - Fool-proof controllability and inherent safety. A measure of the adequacy of the control system design as well as the inherent safety of the reactor by which, in the event of any failure in the system, the system takes up a safe configuration.

X_{414} - Shutdown margin. The minimum shutdown reactivity necessary to provide confidence that the reactor can be made subcritical by means of the control and safety systems starting from any permissible operating condition and that the reactor will remain subcritical without further operator action.

Y_{42} - Basic research. Basic research is conducted to attain a fuller understanding of the subject under study. This needs distinct characteristics of the reactor such as accessibility and flexibility of the core, reactor power stability, ease of start up and shutdown, and rapidity of power level changes.

X_{421} - Core accessibility. The degree of ease in approaching the reactor core from various sides by vertical and horizontal tubes and ports or by other means.

X_{422} - Core flexibility. Core flexibility measures the degree of movability, adaptability, and changeability of the core to rearrangement and to various configurations.

X_{423} - Reactor power stability. A measure of the degree of fluctuation in power or flux. The variation in the flux distribution affects different experiments.

X_{424} - Ease of startup and shutdown. This is the amount of freedom in starting the reactor up at any rate and shutting the reactor down instantaneously to cope with various types of experiments. This depends on the characteristic of the reactor, its power level, and its procedure for safe operation.

X_{425} - Rapidity of power/flux level change. The rapidity in the response time that takes to change the reactor power from one level to another.

Y_{43} - Adaptability for applied research and development. The ability to convert scientific information derived from basic research into technology and practical application. This attribute is to examine different research reactors to carry applied research in science, life science and material testing.

X_{431} - Material testing. Ability to study the radiation effect on the properties of materials. This depends mainly on the energy of neutrons, flux level and exposure time.

X_{432} - Life science research. Ability to study the relation between radiation and cancer and radiation application in medical, crop mutations, insect control, and food preservation. Conducting these types of research depends on the flux level and on the availability of equipments for experiments.

X_{433} - Scientific research. This is to study neutron diffraction, cross section measurements, and nuclear structure and their application. This also depends on the flux level and on the instrumentation availability.

Y_{44} - Services salability. The center provides services to researchers, other universities, organizations and industry. Services include radioisotope production, irradiation services and commercial training programs for utility operators and radiographers. This depends on the facility itself and on the supporting facilities.

X_{441} - Radioisotope productivity. The capability of a particular research reactor to produce artificial radioisotopes for commercial applications. This attribute is a measure of the types and the varieties of the produced radioisotopes and their applications.

X_{442} - Experimental volume availability. The availability of enough irradiation volume inside the reactor to satisfy anticipated commercial needs.

X_{443} - Supporting facility. The size and the variety of facilities which are available to provide and to aid in conducting commercial programs such as radioisotope supporting facilities, hot cells, radiography, and facilities for training utility operators and radiographers.

5.4.5 Definition of compatibility category, attributes, and subattributes

The chance of successfully establishing a nuclear reactor center and helping in the transfer of nuclear technology depends mainly on the following attributes:

(1) availability of specific local resources;

(2) ease of acquisition of imported resources;

(3) community acceptance; and

(4) expandability.

These attributes are divided into more detailed subattributes such as availability of local manpower, ease of maintenance, need of foreign experts, number of research reactor facility systems in use, availability of spare parts, and so on.

Y_{51} - Availability of required local resources. A measure used to evaluate the present status of local resources. This attribute will take into consideration the local human resources for operating the center and on the other hand the degree of ease of maintaining a particular research reactor.

X_{511} - Manpower level of competence. This is an estimate of the existing quality and quantity of the local manpower including personnel available to operate and maintain the center.

X_{512} - Ease of maintenance. Ease in maintaining the reactor system is necessary to decrease the likelihood of any type of failures.

Y_{52} - Ease of acquisition of imported resources. The scope of this attribute is based on the case of acquiring and availability of two major elements: foreign human resources and material resources.

X_{521} - Foreign experts. The minimum number of foreign experts required to ensure the operation of the center to a satisfactory level and to train the local manpower resources for taking over.

X_{522} - Spare parts. To avoid any delay or stoppage in operation, spare parts must be kept on hand. Equipment failures are used as an indication for the number of spare parts that should be kept on hand at each facility to ensure continuous operation.

X_{523} - Number of research reactors in use. The number in use of the reactor type under consideration in the world regardless of their manufacturers.

Y_{53} - Community acceptance. The attitude and the type of reaction expressed toward building the first nuclear research center by the government, public, and educated community.

X_{531} - Educated community acceptance. The degree of acceptance and attitude towards building a nuclear research center and towards the alternative in question.

X_{532} - General public acceptance. The feeling of the general public toward the transfer of nuclear technology to the country and toward each type of research reactor.

X_{533} - Government acceptance. The attitude of the government toward establishing a nuclear research center and preferability. This measure is also an assessment of the willingness of the government to implement the project.

Y_{54} - Expandability. Flexibility in the reactor containment or pool is important so that future requirements for expansion or modification can be met with minimum inconvenience.

6. QUANTITATIVE EVALUATION

6.1. Introduction

Selection of a research reactor facility differs from one case to another due to differences among the needs to be provided. Hence, the proper choice of a research reactor facility is an important requirement to meet the overall goal of the study. In the present case, the overall goal is to enhance Saudi Arabian participation in the peaceful applications of nuclear energy by selecting the most proper research reactor facility to meet the country's needs. The basic characteristics of available research reactor types are reviewed in Chapter 2. In this chapter, four alternatives will be evaluated. The four existing facilities which are selected to represent these alternatives are given in Table 6.1.

Table 6.1. Selection of existing nuclear research reactor facilities

Type	Existing facility	Designation
Pool type research reactor	University of Michigan Ford nuclear reactor	FNR
Light water tank type research reactor	Massachusetts Institute of Technology reactor	MITR
Heavy water tank type research reactor	Georgia Institute of Technology research reactor	GTRR
Pulse type research reactor	University of Wisconsin nuclear reactor	UWNR

A set of categories, attributes, and subattributes is developed and defined in Chapter 5. This set will be used in this chapter to judge the four represented alternatives based upon available data and sources.

6.2. Cost

Cost comparison of various research reactor facilities greatly influences the decision in favor of a facility having the lowest cost. However, estimation of the cost of a nuclear research reactor facility is a very complicated problem. There are numerous factors that influence the estimated cost which is only relevant to specific conditions. As a matter of fact, the factors affecting a nuclear reactor facility cost differ greatly from one imported country to another due to differences in labor rates, availability of building materials, labor availabilities and efficiencies, and other factors. The estimated cost for the attributes and subattributes in the present study should be considered only as a general guide to indicate which type of facility is more attractive economically.

The cost category in this study is broken down into capital cost and operating cost. The capital cost includes total reactor and building cost (hazard evaluation, design, construction, building) and support facilites cost (hot cell, radioisotope laboratories, counting rooms, etc.). The operating cost includes maintenance and operation cost, fuel cost, and staffing and management cost.

The estimated capital cost [63-65] which is updated to December 1981 is given in Table 6.2. The estimated operation cost [66-68] is given in Table 6.3.

Table 6.2. Estimated capital cost (M$)

Designation	Year	Producer price index	Reactor and building cost (M$)	Support reactor facilities cost (M$)
FNR	1955	86.9	3.145	0.555
	Dec., 1981	310.1	11.2228	1.9805
MITR	1956	90.8	5.1	0.9
	Dec., 1981	310.1	17.4175	3.0737
GTRR	1960	95.3	3.825	0.675
	Dec., 1981	310.1	12.4463	2.1964
UWNR	1959	95.3	0.6375	0.1125
	Dec., 1981	310.1	2.0744	0.3661

6.3. Technological Soundness

The development of a nuclear research center begins within the framework of evaluation of current research facilities which can fulfill the national goal for a particular country. The decision maker will face, in addition to various factors, many technological problems in the selection among various facilities. The technological

Table 6.3. Estimated operating cost

Designation	FNR	MITR	GTRR	UWNR
Fuel element cost ($)	8,000	8,000	8,000	15,500
Normal element life-time (MW-day)	774	88.33	720	3,650
$/MW-day	10.336	90.569	11.11	4.247
Maintenance and operation cost (thousand $/year)	164	240	100	42.8
Staffing and management (thousand $/year)	246	360	150	64.2
Number of (shift)	3=100%	3=100%	1=57.7%	1=57.7%
Maintenance and operation (thousand $/shift-year)	94.628	138.48	100	42.8
Staffing and management (thousand $/shift-year)	141.942	207.72	150	64.2

evaluation of the existing research reactors will provide the basis for confident selection of a suitable type. Material availability, proven research reactor type as well as construction time will serve as criteria to evaluate various facilities from technological point of view. Material availability includes fuel, coolant, moderator and reflector availability. Examination of existing alternative facilities and of the experience of the vendors is conducted to identify the commercial viability of each nuclear research reactor facility.

In connection with the selection of a nuclear research reactor, the long-term availability of nuclear fuel services needs to be carefully evaluated for the continuity of the nuclear center operation. The availability of the fuel depends greatly on the percentage of uranium enrichment because of nuclear weapon proliferation concerns [69]. The availability of other reactor materials varies. Light water is the best from an availability point of view. Coolant, moderator and reflector materials as well as fuel enrichment for each represented alternative are given in Table 6.4. The author's judgment of the availability of each material used is given in Table 6.5.

During the last couple of years, construction times have shown an increasing trend, partly because of permit delays but also due to increasing delivery times for key

Table 6.4. The materials used in each alternative

Name of the reactor	Fuel enrichment (%)	Coolant	Moderator	Reflector
FNR	93	H_2O	H_2O	(75%) H_2O + (25%) D_2O
MITR	93	H_2O	H_2O	(75%) D_2O + (25%) H_2O surrounded by graphite
GTRR	93	D_2O	D_2O	(50%) D_2O + (50%) graphite
UWNR	(50%) 70 + (50%) 20	H_2O	H_2O + ZrH	(50%) graphite + (50%) H_2O

Table 6.5. The availability percentage of the reactor materials

Material	Availability (%)
H_2O	100
D_2O	70
H_2O + ZrH	80
Graphite	90

components. Experience also indicates that the first nuclear center project in a non-nuclear country has required more time for construction due to transportation and regulation factors [70]. Table 6.6 shows a relative comparison between the four alternate facilities from a construction time point of view (see Table 6.6).

Table 6.6. Construction time for the four facilities

Name of the reactor	Start of construction	Start of operation	Construction time (year)
FNR	July, 1955	Sep., 1957	2.25
MITR	June, 1956	July, 1958	2.17
GTRR	Oct., 1960	Dec., 1964	4.25
UWNR	- 1959	Mar., 1961	~2

At the present time, there is a number of different types of research reactors in operation in the world, but the choice of a reactor for a country embarking upon its first nuclear research center project may be more limited than this number would indicate. Complex equipments which incorporate new technology, new design or new materials, or which are produced by particular firms for the first time can only be acquired with risk. The risk of production units which are not working to the standards of prototypes, or inexperienced producers not being able to manufacture to specifications. The only protection against such risks is to confine procurement to proven equipment and manufacturers. Judgments of provenness can only be based on actual operating experience with the facility of a particular manufacturer.

The four represented alternatives have been in satisfactory operation and also have reached the proven technology stage. The degree of soundness depends mainly on manufacturers and their experiences with supplying earlier proven facilities. The name and the total number supplied by each vendor of each type, as well as the total existing operating facilities of each alternative [71] are given in Table 6.7.

Table 6.7. Vendors' experiences with nuclear research facilities

Reactor name	FNR	MITR	GTRR	UWNR	Total #
Vendor name	B & W	ACF or AC	GNEC or Comb	GA	--
Total number constructed by vendor	5	3	1	49	58
Total existing operatable facility	33	11	4	51	99
(Vendor/all total) Normalization	0.051	0.030	0.01	0.495	0.586

6.4. Safety Consideration

Safety has been an important consideration from the very beginning of the development of nuclear reactors. Although nuclear reactors of various types and sizes have been built and operated without endangering public safety, the history of nuclear energy and the frequent exposure of the public to various books, popular news media and publications try to convince the public that nuclear energy is not safe. It is a fact that nuclear energy was first used as an atomic bomb for destructive purposes, but it is also a fact that nuclear reactors are inherently safe and differ greatly from a nuclear bomb. It should be emphasized that it is impossible for a nuclear reactor to behave like an atomic bomb, due to basic differences in their design, material, and underlying principles. For this reason, while considering the safety of nuclear research reactor facilities, emphasis is only paid to possible radioactivity release.

Most reactors produce and accumulate large amounts of radioactive isotopes in the fuel material. In adverse circumstances, a reactor may suddenly release an amount of energy which can result in large-scale dispersal of radioactive materials to the environment, creating an extremely hazardous situation [72]. The probability of such an accident can be reduced by introducing safety features in the design of the reactor, its control system and operating the reactor with proper safeguard systems. The consequences of a release of radioactive material, if an accident occurs, can be minimized by proper location of the reactor and design of its building.

An examination of the safety aspects should include an evaluation of the possible hazards to the public, to the reactor personnel, and to the reactor itself, all of which are important in selecting a research reactor facility. Safe operation of a particular reactor depends on reliability of the system taking into account mechanical failures and human errors. Safe operation also depends on the response speed of the control system. Safety considerations concerning the public and reactor personnel include the radiation level and site selection.

Human error and equipment failure, scram insertion speed, and radiation level on reactor top and on general floor area of reactor building [63] are given in Table 6.8.

The flexibility in locating a research reactor facility depends greatly on the type of reactor and on the availability of the land. There are two approaches to the site selection. The first is an exclusion area if sufficient land is available. The second is a gastight building or air-processing system (complete containment) if land is not available. Since the site of the center most probably will

Table 6.8. Safety evaluation of research reactor facilities

Reactor name	Equipment error [Error/(full operation year)]	Human error [Error/(full operation year)]	Maximum time for complete scram (sec)	Radiation level on reactor top (m rem/hr)	Radiation level on general floor area (m rem/hr)	Compatibility with the site (%)
FNR	20.276	2.765	0.45	1	10	40
MITR	21.452	1.430	1.82	1	10	70
GTRR	71.800	10.980	1.21	1	10	70
UWNR	53.571	16.071	1.00	1	10	85

be in a heavily populated city, a gastight building will be employed to contain fission products in case of an accident. Of course, the requirements of the type of containment depends upon the type of reactor to some extent. MITR and GTRR systems are completely sealed in a closed loop cooling system. A tank type reactor takes less area than a pool type reactor and thus it is convenient to construct a gastight containment to cover the reactor area. However, for a pool type reactor, a gastight containment covers a much larger area including the pool and laboratory facilities. In the case of a TRIGA type reactor, a gastight containment is not required [73,74].

From previous discussion, the compatibility of the four reactor alternatives with a populated site is given also in Table 6.8.

6.5. Serviceability

The selection of a research reactor type is a function of many variable objectives. In the existing study, the most important objective is serviceability of a particular research reactor which would be best suited to the needs of the country. Although there is considerable overlapping in the types of research and uses, each reactor has, in general, distinct characteristics which make it better for certain kinds of services. The attributes which determine the best selected reactor type from serviceability viewpoint are as follows:

(1) training and teaching quality;

(2) usefulness in basic research;

(3) adaptability for applied research and development;

(4) services salability.

Discussion of each one of these attributes and subattributes will follow to compare the four reactor facilities from serviceability viewpoint. Training and teaching quality comprises all subattributes that give safety assurances when the facility is used for training and teaching programs. The subattributes include ease of operation, fool-proof, shutdown margin, and simplicity of fuel handling.

Considerable emphasis should be placed on the operation of research reactors. As a training tool, the reactor should be simple enough so that the operation can be clearly demonstrated and understood. The TRIGA reactor is a practical training instrument permitting an inexperienced student and non-technical personnel to operate the reactor [75]. Thus, the TRIGA reactor is the best training reactor among the four alternatives. The pool reactor is considered the second in ranking because the control system is simple and its actions are easy to explain and observe [76]. The heavy and light water tank reactors have more complicated cooling and recombination systems, and therefore they are ranked as number three [76].

The fuel loading facilities and procedures differ considerably from one type to another. Pool reactor is more convenient in refueling and loading than light water tank reactor because the core of a pool reactor is accessible at all times from the top of the pool. With light water tank reactors, of course, the tank cover must be removed for fuel loading. The pool provides several desirable features: it serves as storage area, and it adds flexibility for transfer of highly radioactive fuel elements. In case of heavy water reactor, a closed system must be maintained. For access, the top shield has removable small plugs above each element [12]. The TRIGA reactor shares with the pool reactor many features of fuel loading. The TRIGA core is also submerged in a pool which provides flexibility in refueling [77].

A multitude of safety features are necessary for a research reactor facility which will be used for training and teaching purposes besides other uses. In the present study, the four reactor alternatives are functioning on the fail-safe principle. The principle is based on the fact that electronic and mechanical components of the control system may fail and that reactor operators are fallible. The control system design is based on this principle to assure that the reactor will always be maintained in a safe condition. For example, if electric power fails, electromagnets or clutches release the rods, allowing them to fall by gravity into the core to shut down the reactor. Gravity is backed up by auxiliary spring or hydraulic mechanism that increases insertion speed and assures more positive insertion by preventing rods from binding as they drop [16]. In addition to fail-safe controllability, there are certain inherent safety features basic to each alternative reactor. All four alternative reactors have a negative temperature coefficient in the operating range. The heavy water reactor is considered safer than light water reactors (tank and pool) because it has the same inherent safety features of light water reactors plus a longer neutron generation time. The TRIGA reactor has better inherent safety than light water reactors due to physical property of its unique uranium-zirconium hydride fuel elements. It gives the TRIGA a prompt negative temperature coefficient [18]. In this respect, another important safety measure is shutdown margin. A fundamental requirement in reactor operation is that there must always be sufficient control poison available to bring the reactor subcritical with some margin to spare. This negative reactivity following the trip of the reactor is referred to as the shutdown margin [78]. The shutdown margin value for each reactor is given in Table 6.9.

Conducting basic research with a particular research

Table 6.9. Training quality

Reactor name	Ease of operation (subjective)	Simplicity of fuel loading (subjective)	Fail-safe controllability and inherent safety (%)	Shutdown margin ($\Delta K/K$)
FNR	85	95	82	0.0075
MITR	80	80	80	0.01
GTRR	80	75	95	0.01
UWNR	95	90	90	0.015

reactor is an important attribute to be considered in the research and development (R&D) program of the country. This needs distinct characteristics of a particular reactor such as accessibility and flexibility of the core, reactor power stability, ease of startup and shutdown, and rapidity of power level changes. The pool reactor core has a great degree of accessibility. It is very easy to introduce samples and experiments through the water from the top of the pool to be placed near or in the core which permits using the highest possible flux. If it is necessary to prepare a complex experiment, it is possible to drain one-half of the pool and to prepare the experiment in the dry half [76]. The pool reactor core is more accessible at all times from the top than the light water tank reactor core. The tank cover must be removed to insert samples or experiments near the core. The closed system and the need for preventing contamination of the heavy water make the core of this type of facility less accessible than in the case of light water reactors (pool and tank). Therefore, all experimental facilities must be designed and built into the system when the facility is constructed. The TRIGA reactor is the same as the pool type reactor. The TRIGA core is accessible from the top of the pool and permits the insertion and removal of samples during full power operation [75].

The flexibility of the core to move and to rearrange its configuration differs from one reactor type to another. Pool and TRIGA reactor cores can be changed to various configurations more easily than tank reactor core (light or heavy water). In addition, the pool reactor core can be moved and operated in several positions in the pool. In contrast, the tank and TRIGA core are stationary, which reduces the flexibility to move the core freely.

It is not enough for conducting research programs to have just access to the reactor core, but operational characteristics such as operating stability, ease of start-up and shutdown and rapidity of power change of the reactor are also important subattributes. The light water reactors (pool, tank and TRIGA) and heavy water tank reactor are quite stable in operation. Automatic control will hold the power within ±1% of the desired value in case of pool, light and heavy water tank reactors. But in case of TRIGA (UWNR) reactor, the automatic control system will hold the power within ±5% of the desired value [12,79]. However, temperature changes from day to day influence the regulating rod positions somewhat, which causes variations in the flux distribution in all four reactor types. Generally, these changes in flux distribution and the effect on the experiments are small and within acceptable limits. A long term change in the position of the control rods and in flux distribution is caused by burnup of fuel. Sharp changes in

Table 6.10. Accessibility and flexibility of the core and reactor power stability

	Core accessibility (%)	Core flexibility (%)	Reactor power stability			
			Automatic control system error (%)	Normal average burnup (fission/cc) x 10^{20}	Normal average reactivity addition rate $\Delta k/k/\text{min}$	Final judgment (%)
FNR	95	95	±1%	3.05	0.004	80
MITR	70	75	±1%	10.1	0.00057	70
GTRR	60	75	±1%	4.67	0.005	70
UWNR	95	90	±5%	1.5	0.0035	75

flux distribution occur when fuel elements added or replaced which may occasionally affect an experiment [12]. Accessibility and flexibility of the core and power stability are given in Table 6.10 for each reactor alternative.

Startup and shutdown are easily accomplished in all four reactors. Each reactor can be started up at any rate considered safe. Likewise, they can be shut down almost instantaneously. The heat removal and the heat capacity of the system will take care of the heat produced by fission products after shutdown. However in case of heavy water tank reactor, at power higher than 4 MW, the fission-product heat after shutdown becomes significant, and hours of cooling may be required [12,16].

Rapidity of power change from one level to another differs from one research reactor to another due to differences in their control systems. During operation, large adjustments of power level are made with shim rods. In this study, the four reactors use one set of control rods for both safety and shimming purposes and usually designate these rods as shim-safeties [63]. Ease of startup and shutdown and the normal (average reactivity addition rate) which measures the rapidity of power change are given in Table 6.11 for each represented reactor.

Applied research is to convert scientific information derived from basic research into technology and practical

Table 6.11. Ease of startup and shutdown, and power change rapidity

	Startup and shutdown	Power change rapidity
	Worst case elapsed time from shutdown to coolant independence without fuel distortion	Normal average reactivity addition rate ($(\Delta k/k)$/min)
FNR	0	2.8×10^{-3}
MITR	0	3.7×10^{-3}
GTRR	8 hours	13×10^{-3}
UWNR	0	19×10^{-3}

usage. Nuclear research reactors make a variety of applied research experiments possible in the field of material testing, life sciences, physics and chemistry. So many applied research experiments can be adapted to fit an available reactor, whatever its type, but certain experiments, because of flux requirements or other requirements, can be conducted more easily in certain types of research reactors. Table 6.12 lists the fluxes required for some applied research [80].

The use of nuclear reactor radiation in the study of materials has an important role in nuclear technology and in reactor development research. Interactions of nuclear radiation with matter may be defined as that in which the radiation is sufficiently energetic to produce atomic displacements which change the properties of the medium. The

Table 6.12. Neutron flux required for applied research

Research	Approximate neutron flux required (n/cm^2 sec)
Material Testing . Radiation damage . Long time burn up	$10^{12} - 10^{14}$
Life Science . Medical . Biological . Agricultural	$10^{10} - 10^{12}$
Science (Physics, Chemistry, Nuclear) . Neutron diffraction . Cross section measurements . Chemistry	$10^{8} - 10^{13}$

damage may appear as changes in thermal conductivity, changes in density, corrosion, etc. Another important use for the research reactor is to test various reactor components such as fuel element material due to long time burn up. In general, these types of research require high fluxes and the research reactors with high flux are best suited to a large portion of this research [81].

Research reactors have been widely used in medical, biological and agricultural research. Agricultural research includes mutation breeding which speeds up the development of new plant and animal species, food irradiation so that perishable products may be stored longer, and the study of the behavior of radioactive nuclides in the food chain--soil, plant, animal, in the framework of the agriculturally permissible contamination of soil, water and air. The principal uses of neutrons in medical and biological studies are to produce the biological effects of radiation, and to produce radioactive forms of the tissue constituents. A rapid growth in the use of radiation for cancer therapy has taken place in recent years. The effectiveness of the treatment is based upon two characteristics of thermal neutrons (1) production of little ionization and can pass through tissue with relatively little effect; and (2) interaction with B^{10}, which can be absorbed selectively in cancerous tissue, to produce alpha particles that cause intense local ionization which is very destructive to the diseased tissue [82].

The probability of interactions of neutrons with nuclei is expressed in terms of a "cross section". There are three basic techniques for measuring cross sections: beam or transmission, in pile, and activation. In the beam method, a parallel stream of neutrons is brought out of the reactor, passed through a specimen, and the neutron intensity is measured. By measuring intensity with and without the specimen in place, the total or removal cross section of the specimen can be determined. In the in pile method, material for which the cross section is desired is placed in the reactor, and its effect on the reactivity is measured. This permits an evaluation of the absorption cross section alone, since the neutron balance in the core is affected primarily by the disappearance of a neutron rather than by a change in direction of a neutron. The activation method consists of irradiating a specimen for a measured amount of time, then determining the radioactivity induced by neutron absorption. The activation cross section may be only a part of the entire absorption cross section, since some neutron absorption may produce a stable isotope and, consequently, no radioactivity.

The nuclear research reactor is also useful in the field of physics for investigating atomic displacements in solids caused by neutrons and gamma rays, and analyzing crystal structure by neutron diffraction, scattering, and depolarization techniques. In chemistry, the reactor is used in many research areas, including the effects of radiation on chemical systems, the chemistry of radioactive elements, and the study of chemical reactions.

Conducting research in material testing and science depends greatly upon the flux level and the available facilities such as medical therapy room, greenhouse, neutron spectrometer and other facilities. The evaluation of each alternative will depend upon the flux level and the facilities available in each center to conduct a specific research type. This is shown in Table 6.13.

Services salability factor is part of an integrated research, development and application program of a nuclear research center. Clearly, although important, sale of services by itself does not justify the construction and operation of a research reactor. The multi-purpose use of a research reactor is typical and is the actual situation in most nuclear centers. It is also clear that the type of reactor, size of reacting medium inside the reactor, and supporting facilities will determine to a large extent the possibilities to provide services that would generate revenues to offset some of the operation cost. Services include radioisotope production, irradiation services, and training program for utility operators from neighboring countries and radiographers. Actually, the service plan may require extended operation and thus incur further operation expenditures.

The production of radioisotopes plays a significant role in the utilization of the reactor. The application of radioisotopes has been widely used in industry, agriculture, scientific research and medicine. The radioisotope production capacity of the research reactor is usually governed by other various uses of the reactor. Radioisotope production requires a certain degree of continuous running and regularity of reactor cycle. Further, it should be recognized that

Table 6.13. Conducting applied research in research reactors

	Material testing	Life science			Science		
	Max flux (in core) n/cm^2 sec	Flux n/cm^2 sec	Facility	Rating (sub)	Flux n/cm^2 sec	Facility	Rating (sub)
FNR	3×10^{13}	$>10^{12}$	Greenhouse 10,000 Ci Co - 60 source	80	$>10^{13}$	4 single axis 1 multiple spectrometers 4420 + 6600 multichannel analyzers	85
MITR	3×10^{13}	$>10^{12}$	Medical therapy room 4,000 Ci Co - 60 source	80	$>10^{13}$	4 single axis 2 triple axis spectrometers 4096 multichannel analyzers	80
GTRR	6.5×10^{13}	$>10^{12}$	Bio-medical room 55,000 Ci Co - 60 source	90	$>10^{13}$	2 multiple axis spectrometers 1024 + 4096 multichannel analyzers	80
UWNR	8×10^{12} (pulsing) 6×10^{16} (3.7 mesec period)	$>10^{12}$	None Spent fuel as source	65	8×10^{12}	None in operation spectrometers 4096 multichannel analyzers	65

one type of reactor is not ideal for the production of all radioisotopes. The production of high specific activity isotopes or high intensity sources, Co - 60, C - 14 or tritium, creates a demand for high fluxes, e.g. 10^{15} n/cm^2 sec or even higher. It is therefore common for research reactor centers to concentrate on the production of short-lived radioisotopes [83, 84]. In our case of study, the principal isotopes produced by the four research reactor centers, the half-life and the use of each isotope, and accordingly the rating of each reactor center are given in Table 6.14.

Almost all four reactors have some means of inserting samples into or close to the reactor core. All research reactors have some means of getting a beam of neutrons out of the reactor core. However, it is not enough to have access to the reactor core to enhance salability of services, but the experimental volume available inside the reactor to accommodate various experimental programs is an important subattribute in evaluation of alternatives. Table 6.15 gives dimensions and description of beamports, pneumatic tubes, in pool and thermal column, for each reactor.

The variety and the size of the supporting facilities are important subattributes for measuring the potential of providing various services such as radioisotope production, irradiation services and training of utility operators and radiographers. Table 6.16 shows the facilities available and

Table 6.14. Principal isotopes produced and their uses

	Isotope	Half-life	Medical application	Industrial application	Agricultural and hydrological application
FNR Rating 80%	Fluorine F-18	109.8 m	Bone scanning		
	Chlorine Cl-36	$3.1x10^5$ y			Soil, plant and animal nutrition; water movement
	Bromine Br-80	17.6 m		Leak detection	
	Bromine Br-82	35.34 h	Fluid volume	Tracing	Surface water (discharge measurement), ground water tracing
	Iodine I-131	8.05 d	Blood cell labelling; Thyroid function	Tracing	Soil, plant and animal nutrition; ground water direction
	Cesium Cs-134m	2.9 h			
MITR Rating 70%	Molybdenum Mo-99	67.0 h	Liver scanning		Soil, plant and animal nutrition
	Dysprosium Dy-165	2.32 h			
	Osmium Os-191	15 d			
	Gold Au-198	64.8 h	Cerebral blood flow; Liver function	Tracing	Entomology; water stream gauging (surface water); ground water velocity

Table 6.14. Continued

	Isotope	Half-life	Medical application	Industrial application	Agricultural and hydrological application
GTRR Rating 70%	Fluorine F-18	109.8 m	Bone scanning		
	Sodium Na-24	15.0 h	Cerebral blood flow; circulatory studies; sodium metabolism	Tracing	Soil, plant and animal nutrition; water stream gauging
	Yttrium Y-90	64 h	Radiotherapy	Thickness gauging	
	Lanthanum La-140	40.22 h			Sediment transport
UWNR Rating 65%	Fluorine F-18	109.8 m	Bone scanning		
	Sodium Na-24	15.0 h	Cerebral blood flow; circulatory studies; sodium metabolism	Tracing	Soil, plant and animal nutrition; water stream gauging
	Copper Cu-64	12.8 h	Copper metabolism; gastro-intestinal tract studies; Wilson's disease		

Table 6.15. Experimental volume available

	FNR		MITR	
	Description	Dimensions (cm)	Description	Dimensions (cm)
Beamports	2 horizontal	20.3 Dia	6 radial	11.43 Dia
			4 radial	16.83 Dia
			1 radial	32.23 Dia
	8 horizontal	15.2 Dia	1 tangential	11.43 Dia
			1 tangential	16.83 Dia
Pneumatic tubes	4 on west core face	2.54 Dia	4 in graphite reflector	2.54 Dia
			1 in heavy-water reflector	4.13 Dia
Reactor core in-core	Central and peripheral irradiation locations	2.54 x 7.62	2 with helium cover	4.45 Dia
			1 with temp. control	2.86 Dia
			Additional space	6.86x5.08
Reactor pool (in-pool)	South core face and pool locations	30.5 x 30.5	None	--
Thermal column	Inoperative	--	1 capsule	4.13 Dia
			1 graphite-lined "hohlraum"	183 x 183
Equivalent diameter without thermal column	62.777 Dia		59.199 Dia	
Rating (%)	85%		80%	

GTRR		UWNR	
Description	Dimensions (cm)	Description	Dimensions (cm)
1 horizontal 8 horizontal 1 horizontal	15.24 Dia 10.16 Dia 5.08 x 15.24	4 horizontal	15.2 Dia
2	2.54 Dia	1	3.81 Dia
3 lattice position 2 heavy water 2 heavy water 10 graphite 4 graphite	6.67 Dia 8.57 Dia 14.0 Dia 10.16 Dia 15.24 Dia	3 hydraulic irradiation tubes 1 hydraulic irradiation tube	6.35 Dia 2.5 Dia
None	--	3 irradiation baskets	7.6 x 7.6
South face	45.7 x 45.7 or 15.24x15.24	1	102 x 102
61.6625 Dia		35.868 Dia	
82%		60%	

Table 6.16. Support facilities

	Radioisotope laboratories		Hot cell		Neutron radiography		Utility operator training programs	Rating (%)
	Description	Area m^2	Description	Area (m)	Description	Dimensions cm		
FNR	10 laboratories	186	2 cells One connected to the pool	1.83 x 2.44	One One	7.62 Dia 20.32 x 25.4	Being conducted	85%
MIT	14 laboratories	--	1 cell 1 cell	1.2 x 1.2 1.2 x 1.8	One	15.2 Dia		80%
GTRR	10 laboratories	2230	1 cell	6.4 x 2.1	1 1 1	10.16 Dia 10.16 Dia 15.24 Dia	Being conducted	90%
UWNR	3 laboratories	305	1 cell	2.4 x 2.4	Under development	--	Being conducted	75%

their size for each reactor.

6.6. Compatibility of Nuclear Transfer

The chance for successful implementation of establishing a nuclear research center depends greatly on promoting the transfer of skills and knowledge related to nuclear energy to the recipient country, on the efforts made by the country in utilizing the local resources to carry out its nuclear energy activities safely and more efficiently, and on the community acceptance of the role of the nuclear center in the country. The mixture of domestic and imported resources and community acceptance are very important factors in the evaluation of the present research reactor types in a most convenient way for the interest of the country.

Availability of required local resources is an attribute to evaluate the present status of the local resources taking into consideration the local human capital for maintaining the research reactor safely and efficiently and the degree of ease of maintenance of a particular research reactor. The number of the present local nuclear engineers who have had experimental applied courses in each reactor type is given in Table 6.17. Table 6.17 indicates the limitation of local manpower which is the major constraint on the transfer of nuclear technology. A great effort should be spent to provide an adequate local skilled manpower by initiation of intensive training programs within the country and

Table 6.17. Ease of maintenance and the experience of the local manpower with each reactor

	Local manpower			Ease of maintenance (%)
	Graduate	Under graduate	Total	
FNR	4	11	15	90
MITR	4	-	4	80
GTRR	-	6	6	60
WUNR	1	-	1	95

abroad to meet the requirements of the center on schedule. The training programs would include short nuclear courses in classrooms and laboratories, training on reactor simulator, acquaintance with actual operation of research reactor similar to the one which will be constructed in the country, and finally training during the construction of the center and on the job [85].

Ease of maintaining the reactor system differs from one research reactor to another due to differences in their designs. The TRIGA has the simplest system design. The heavy water tank reactor requires a complex close system to prevent loss or contamination of the heavy water. Pool and light water reactors have almost the same degree of ease in maintaining the system [12, 75]. The rating of each reactor according to ease of maintenance is also given in Table 6.17.

Experience history with a particular research reactor facility in the world, the minimum number of required foreign experts to aid in operating the center and training the local personnel, and the estimated number of spare parts for each reactor type represent the main factors in acquisition of nuclear technology successfully. The increased number of users of a particular research reactor facility in the world at the present time is not only advantageous to facilitate the acquisition of this generation of reactor from various exporting countries, but also it helps in acquisition of know-how and in transferring different experiences from different centers in the world which had similar situation. This aids in adapting an efficient scheme for the center avoiding the obstacles faced by other countries. The population of each reactor type regardless of their manufacturers is shown in Table 6.18. The name, location, designation, contractor, power, and the startup date of each reactor are given in Appendix A.

Operation of the nuclear research center requires a certain level of qualification, training, and experience. To overcome the lack of these skills in the local manpower, foreign high quality skills should be attracted to transfer the nuclear knowledge. The qualifications of foreign experts should include high technological skill, ability to work with others, and ability to adapt to a different

Table 6.18. Acquisition of imported resources

	Number of research reactor facilities in the world			Required staffing				Spare parts
	USA	Abroad	Total	Scientific technical	Operations	Support	Total	Equipment failure rate (#/year)
FNR	13	20	33	5	10	17	32	13.2
MITR	8	3	11	5	12	9	26	12
GTRR	2	2	4	5	7	12	24	17
UWNR	25	26	51	1	5	2	8	6

environment [86]. The number of specialists who are running each reactor facility successfully at the present time is given in Table 6.18.

Spare parts and machine components are not manufactured locally at the present time. As a result, difficulties in getting spare parts are expected to be one of the problems in implementation of the nuclear center. The center must import more spare parts including some large items to avoid any delay or stoppage in operation. Placing orders, shipping, clearance requirements and other problems of spare parts should be overcome by prior arrangements, a reliable management program, and an efficient maintenance program. The rate of equipment failure per year for each reactor facility is used as an indication for the number of spare parts that should be kept on hand. These rates are shown in Table 6.18.

The need for nuclear application in Saudi Arabia becomes very important to raise the stage of development of the country from scientific and technological viewpoint. There is no doubt that the Saudi community is very aware of its shortage regarding application of nuclear energy in medicine, food, industries, agriculture, etc., but the community acceptance will play a very strong role and effect to a large extent the establishment of nuclear research center in the country. The attitude of the government, public, educated people is an important basis for starting a nuclear program.

In industrially developed countries, organized opposition to nuclear energy has grown and become an increasingly painful phenomenon, especially after the Three Mile Island accident. With respect to Saudi Arabia, the situations is entirely different. There is no organized anti or pro nuclear group in the country, because it is irrelevant to the nature of the society. The community, in general, trusts scientists, engineers, and professional people and the information and facts that are presented by them. This can provide the right view and answer about the concern of the community regarding safety, environment, and health.

While complete acceptance is not likely, the government, public, and educated people have a favorable attitude toward nuclear technology in general. Saudi government has made long range projections on nuclear technology to be achieved. In Table 6.19 the expected degree of government acceptance in regard to each reactor type is stated [87, 88]. The public and educated people attitude toward nuclear technology has been assessed by Kusayer [89], and the summarized results are represented in Table 6.20. The public attitude toward each reactor type is the same. Six Saudi nuclear graduate students studied the four reactor types and their attitude about each reactor type is given in Table 6.19.

Table 6.19. Community acceptance

	Educated people attitude	Public attitude	Government attitude
FNR	75	60	80
MITR	60	Same	60
GTRR	50	Same	55
UWNR	90	Same	75

The ability to introduce modification to the reactor containment or pool is an attribute that needs to be considered to ensure future expansion easily. The degree of flexibility to carry out this modification differs from one reactor type to another [90]. Pool and TRIGA types have more flexibility than tank reactor types. In case

Table 6.20. Attitude of public and educated people toward nuclear technology

Attitude	Public (%)	Educational level		
		Graduate student	Under-graduate student	High school student
In favor	35.48	41.86	53.42	58.06
Against	29.03	37.21	31.51	17.20
Indifferent	18.35	18.60	13.01	18.28
No opinion	16.13	2.33	2.05	6.45

the pool and TRIGA, the modification can be done by removing one face and expanding the other faces of the pool, but in case of a tank reactor type, almost all faces should be expanded. However, modification of light water tank is easier than heavy water tank because heavy water system is more complex and should be closed to prevent loss of the heavy water. The degree of expandability is shown in Table 6.21 for each reactor type.

Table 6.21. Degree of expandability for each reactor type

Reactor type	Expandability (%)
FNR	85
MITR	60
GTRR	50
UWNR	85

7. APPLICATION OF MULTIATTRIBUTE UTILITY DECISION FOR ALTERNATIVES EVALUATION

7.1. Introduction

The categories, attributes, and subattributes were selected to provide a comprehensive and realistic variable which will be used to measure between the alternative research reactor facilities and to which extent that each alternative will satisfy the overall objective. The value of these categories, attributes, and subattributes corresponding to the level of impact of each alternative was determined by the characteristics of each facility as discussed in Chapter 6. The selected categories, attributes, and subattributes were measured on different scales. However, there are some qualitative attributes and subattributes which reflect the intangible objectives; the subjective scale was used to rank the level of performance of each alternative. The quantified selected units were based on available information and the ease of quantification of such information. Table 7.1 summarizes the selected categories, attributes, subattributes, their units, and the level of impact associated with each alternative as discussed in more detail in Chapter 6.

In this chapter, the multiattribute utility function will be applied to evaluate between the four alternative facilities following these steps:

Table 7.1. Summary of the level of impact of each alternative

Symbol	Category/attribute/ subattribute	Unit	FNR	MITR	GTRR	UWNR
Z_1	Cost					
Y_{11}	Capital cost					
X_{111}	Reactor and building cost	M$	11.223	17.418	12.446	2.074
X_{112}	Supporting facility cost	M$	1.981	3.074	2.196	0.366
Y_{12}	Running cost					
X_{121}	Fuel cost	$/MW-day	10.336	90.569	11.11	4.247
X_{122}	Maintenance and operation cost	10^5 $/shift-yr	0.946	1.385	1.00	0.428
X_{123}	Staffing and management cost	10^5 $/shift-yr	1.419	2.077	1.50	0.642
Z_2	Technological soundness					
Y_{21}	Availability of material					
X_{211}	Fuel	Enrichment	93	93	93	45
X_{212}	Coolant	%	100	100	70	100
X_{213}	Moderator	%	100	100	70	80
X_{214}	Reflector	%	92.5	83.75	80	95
Y_{22}	Construction time	yr	2.25	2.17	4.25	2

Table 7.1. Continued

Symbol	Category/attribute/subattribute	Unit	FNR	MITR	GTRR	UWNR
Y_{23}	State-of-the-art level	$\frac{\text{R.build by vendor}}{\text{all total}}$	0.051	0.030	0.010	0.495
Z_3	Risk					
Y_{31}	Reliability of operation					
X_{311}	Equipment error	$\frac{\text{Error}}{\text{Operation year}}$	20.276	21.452	71.800	53.571
X_{312}	Human error	$\frac{\text{Error}}{\text{Operation year}}$	2.765	1.430	10.980	16.071
X_{313}	Scram insertion time	sec	0.45	1.82	1.21	1.00
Y_{32}	Radiation level					
X_{321}	Radiation on reactor top	Mrem/hr	10	10	10	10
X_{322}	Radiation on general floor area	Mrem/hr	1	1	1	1
Y_{33}	Compatibility with selected site	%	40	70	70	85
Z_4	Serviceability					
Y_{41}	Training and teaching quality					
X_{411}	Ease of operation	Subjective	85	80	80	95
X_{412}	Simplicity of fuel loading	Subjective	95	80	75	90

Table 7.1. Continued

Symbol		Category/attribute/subattribute	Unit	FNR	MITR	GTRR	UWNR
	X_{413}	Fool-proof controllability and inherent safety	%	82	80	95	90
	X_{414}	Shutdown margin	$\Delta k/k$	0.0075	0.01	0.01	0.015
Y_{42}		Usefulness in basic research					
	X_{421}	Core flexibility	%	95	70	60	95
	X_{422}	Core flexibility	%	95	75	75	90
	X_{423}	Reactor power stability	%	80	70	70	75
	X_{424}	Ease of startup and shutdown	hr	0	0	8	0
	X_{425}	Rapidity of power/flux level change	$10^{-3}(\frac{\Delta k/k}{min})$	2.8	3.7	13	19
Y_{43}		Adaptability for applied research and development					
	X_{431}	Material testing research	$10^{13}(\frac{n}{sec\ cm^2})$	3	3	6.5	0.8
	X_{432}	Life science research	Subjective	80	80	90	65
	X_{433}	Science research	Subjective	85	80	80	65
Y_{44}		Services salability					
	X_{441}	Radioisotope productivity	Subjective	80	70	70	65

Table 7.1. Continued

Symbol	Category/attribute/ subattribute	Unit	FNR	MITR	GTRR	UWNR
X_{442}	Experimental volume availability	Subjective	85	80	82	60
X_{443}	Supporting facility	Subjective	85	80	90	75
Z_5	Compatibility with Saudi Arabia					
Y_{51}	Availability of required local resources					
X_{511}	Local manpower	Persons	15	4	6	1
X_{512}	Ease of maintenance	%	90	80	60	95
Y_{52}	Ease of acquisition of imported resources					
X_{521}	Foreign experts	Persons	32	26	24	8
X_{522}	Spare parts	Failure/yr	13.2	12	17	6
X_{523}	Generation of reactor facility	#	33	11	4	51

Table 7.1. Continued

Symbol		Category/attribute/ subattribute	Unit	FNR	MITR	GTRR	UWNR
Y_{53}		Community acceptance					
	X_{531}	Educational people	Subjective	75	60	50	90
	X_{532}	Public	Subjective	60	60	60	60
	X_{533}	Government	Subjective	50	30	25	45
Y_{54}		Future expandability	%	85	60	50	85

(1) adopting an acceptable range (worst and best value) for each category, attribute, and subattribute;

(2) verifying independence (utility and preference) among categories, attributes in each category, and subattributes in each attribute, as perceived by the decision maker;

(3) finding the utility function, through the lottery procedure, for each category, attribute, and subattribute which accurately reflects the decision maker's judgment; and

(4) assessing the scaling constants which represent probabilities trade-off among categories, attributes in each category, and subattributes in each attribute.

Therefore, this proposed approach would be to develop the multiattribute utility (MAU) function technique which is based on the theoretical background described in Chapter 4. The MAU technique will provide an optimization of the overall utility of each of the four alternatives. The alternative giving the highest utility value is the optimal alternative to choose since this can be viewed as yielding the highest utility for the decision maker.

A computer program is developed to assist in performing a utility analysis. The program calculates the following:

(1) the coefficients for each category, attribute, and subattribute which represent the best fit for an exponential curve;

(2) the values of the ks, scaling constants, which are needed to construct the multiattribute utility function; and

(3) the utility function for each alternative.

The program is developed with a great degree of flexibility so that it can be used for solution of a variety of decision problems of the same nature as the problem analyzed here.

7.2. The Range of Categories, Attributes and Subattributes

Since categories, attributes, subattributes have been selected and their level of impact has been estimated for each alternative, an acceptable range for each of them should be estimated. The range for each of them is selected in a way to include the worst and best possible levels. The limits are intended to be realistic to facilitate the synthesis of the utility function through the use of lotteries. Also, the two limits should reflect realistic experience rather than very infrequent maximum or minimum values that might occur, and the two limits should also depend on how much the decision maker or the planner would allow each of the categories, attributes, and subattributes to vary for a particular plan.

In the present case, the selected limits are listed in Table 7.2, and their selection is based upon the data and discussion in Chapter 6.

7.3. Verification of the Independence of Decision Variables

The categories, attributes, and subattributes are clearly defined in Chapter 5 and their limits and units are specified in the last section. Then, the first step in the assessment of the multiattribute utility function is to verify the nature of interdependence among the categories, attributes in each category, and subattributes in each attribute as perceived by the decision maker. It must be mentioned now that these verifications are subjective and may vary from person to person. The preferences may also vary with time. The definitions in Chapter 5 for each category, attribute, and subattribute and applicable to the present study, aid to a great extent in the verification of the decision variables. The two sets of variables whose independence are needed to be verified are "preferential independence" and "utility independence".

Preferential independence means that the trade-offs between any

(1) two categories, through both attributes and subattributes, do not depend on the level of the

Table 7.2. Worst, best, and certainty equivalent for each subattribute

Symbol			Category/attribute/ subattribute	Worst	Best	Certainty equivalent
Z_1			Cost			
	Y_{11}		Capital cost			
		X_{111}	Reactor and building cost	50	2	40
		X_{112}	Supporting facility cost	10	0.2	8
	Y_{12}		Running cost			
		X_{121}	Fuel cost	100	1	30
		X_{122}	Maintenance and operation cost	2.5	0.2	1.8
		X_{123}	Staffing and management cost	3	0.3	2.5
Z_2			Technological soundness			
	Y_{21}		Availability of material			
		X_{211}	Fuel	95	10	25
		X_{212}	Coolant	0	100	65
		X_{213}	Moderator	0	100	65
		X_{214}	Reflector	0	100	65
	Y_{22}		Construction time	10	2	4
	Y_{23}		State-of-the-art-level	0	0.586	0.2
Z_3			Risk			
	Y_{31}		Reliability of operation	100	10	20
		X_{311}	Equipment error	100	10	20
		X_{312}	Human error	50	1	5
		X_{313}	Scram insertion time	3	0.2	1
	Y_{32}		Radiation level			
		X_{321}	Radiation on reactor top	25	0	5
		X_{322}	Radiation on general floor area	5	0	1
	Y_{33}		Compatibility with selected site	0	100	70

Table 7.2. Continued

Symbol	Category/attribute/ subattribute	Worst	Best	Certainty equivalent
Z_4	Serviceability			
Y_{41}	Training and teaching quality			
X_{411}	Ease of operation	50	100	60
X_{412}	Simplicity of fuel loading	50	100	70
X_{413}	Fool-proof controllability and inherent safety	50	100	65
X_{414}	Shutdown margin	0.001	0.05	0.008
Y_{42}	Usefulness in basic research			
X_{421}	Core accessibility	40	100	50
X_{422}	Core flexibility	30	100	45
X_{423}	Reactor power stability	40	100	70
X_{424}	Ease of startup and shut-down	8	0	2
X_{425}	Rapidity of power/flux level change	1	35	10
Y_{43}	Adaptability for applied research and development			
X_{431}	Material testing research	10^{-2}	10	10^{-1}
X_{432}	Life science research	50	100	60
X_{433}	Science research	50	100	60
Y_{44}	Services salability			
X_{441}	Radioisotope productivity	50	100	65
X_{442}	Experimental volume availability	50	100	65

Table 7.2. Continued

Symbol			Category/attribute/ subattribute	Worst	Best	Certainty equivalent
		X_{443}	Supporting facility	50	100	65
Z_5			Compatibility with Saudi Arabia			
	Y_{51}		Availability of required local resources			
		X_{511}	Local manpower	0	35	20
		X_{512}	Ease of maintenance	50	100	55
	Y_{52}		Ease of acquisition of imported resources			
		X_{521}	Foreign experts	35	5	20
		X_{522}	Spare parts	25	5	10
		X_{523}	Generation of research reactor facility	2	60	10
	Y_{53}		Community acceptance			
		X_{531}	Educational people	0	100	40
		X_{532}	Public	0	100	50
		X_{533}	Government	0	100	35
	Y_{54}		Future expandability	40	100	75

other categories, or

(2) two attributes, through both subattributes, do not depend on the level of other attributes in the same category, or

(3) two subattributes do not depend on the level of other subattributes in the same attribute.

As an illustration, the two categories (serviceability (Z_4) and compatibility of nuclear transfer to Saudi Arabia (Z_5)) can be shown to be preferentially independent of other categories (cost (Z_1), technological soundness (Z_2), and risk (Z_3)). The following outcome is true for the decision maker:

$(Z_1^1, Z_2^1, Z_3^1, \bar{Z}_4, \bar{Z}_5) \lesssim (Z_1^1, Z_2^1, Z_3^1, Z_4^1, Z_5^0)$ (where Z^1 and Z^0 are in the best and worst level, respectively) or

$(10, 100, 0, 25, 30) \lesssim (10, 100, 0, 100, 0)$.

This implies that the following is true too:

$(Z_1^0, Z_2^0, Z_3^0, \bar{Z}_4, \bar{Z}_5) \lesssim (Z_1^0, Z_2^0, Z_3^0, Z_4^1, Z_5^0)$

or

$(100, 0, 100, 25, 30) \lesssim (100, 0, 100, 100, 0)$.

Since this is true for the decision maker along with similar implication for different levels for (Z_4,Z_5) and (Z_1,Z_2,Z_3), it can be concluded that (Z_4,Z_5) is preferentially independent of (Z_1,Z_2,Z_3). By going through identical procedures, the preferential independence hopefully is true for the categories, attributes, and subattributes in this present case.

Utility independence means that the preference order for lotteries involving only changes in the levels of attribute (category, subattribute) does not depend on the levels at which the other attributes (categories, subattributes) are held fixed. Consider two options, each presenting a lottery involving X_{111} (reactor and building cost) with different probabilities. Option I gives the best value of X_{111}, $X_{111}^{1} = 2$ M\$, with probability $p = 0.60$; while option II gives X'_{III} with $p = 0.40$. Set all other subattributes in the same attribute at their best levels which is one in this case (X_{112}). Since less reactor and building cost is desirable, option I is preferred to option II, as shown in Figure 7.1.

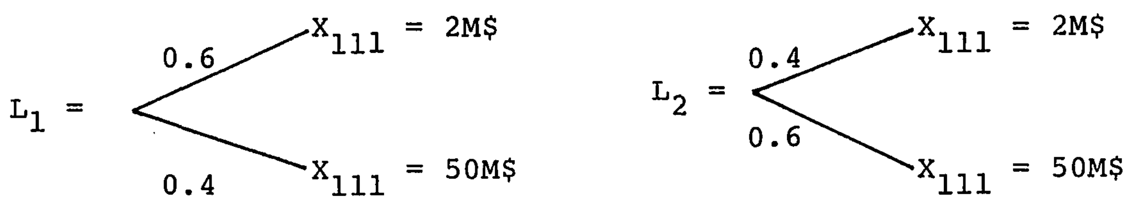


Figure 7.1. Verifying the utility independence

Changing the value of subattribute (X_{112}) to its worst level does not change the preference order between the lotteries as indicated in Figure 7.1. Repeating this procedure using other levels of X_{111}, X_{112}, and p does not change the preference order. Hence, X_{111} is utility independent of X_{112}. Going through the categories, attributes, and subattributes by the same procedure we hope that all of them are utility independent of each other.

Using such independent notions, a multiattribute utility function can be decomposed into parts as follow:

$$\mu(x_1,\ldots,x_i) = \sum_{i=1}^{n} K_i \mu_i(x_i) \quad \text{if } \sum k_i = 1 \tag{1}$$

or

$$1 + K\mu(x_1,\ldots,x_i) = \prod_{i=1}^{n} [1 + Kk_i \mu_i(x_i)] \text{ if } \sum k_i \neq 1 \; . \tag{2}$$

Usually $\sum k_i$ does not equal one unless the values of k_is are normalized.

Equation 1 is the additive utility function, and equation 2 is the multiplicative utility function where K is as follows:

$$K + 1 = \prod_{i=1}^{n} (Kk_i + 1) \quad . \tag{3}$$

7.4. Assessment of Individual Utility Functions

The actual assessment process requires personal interaction, since the utility function of the decision maker is a formalization of his subjective preferences. To capture the decision maker's preferences, a 50-50 chance lottery is used. This procedure permits the internal structuring of each attribute scale in such a manner that it meshes externally with the other scales. In conjunction with applying the midvalue splitting technique, we may ask questions to determine in a qualitative way, the general form of each utility function, e.g., whether it is concave, convex, or linear.

In Table 7.2, where the range for each category, attribute, and subattribute is chosen, the utility function will be scaled so that

$$\mu(\text{best}) = 1$$
$$\mu(\text{worst}) = 0 \; .$$

Thus, for subattribute X_{121} (fuel cost)

$$\mu(\text{best}) = \mu(1 \text{ \$/MW day}) = 1$$

and

$$\mu(\text{worst}) = \mu(100 \text{ \$/MW day}) = 0 \; .$$

Now, the two endpoints are determined for developing the graph of X_{121}. To specify additional points on the curve, the certainty equivalent of some simple lotteries should be found involving different levels of the subattributes. Thus, the certainty equivalent value of X_{121} which has a utility of 0.5 is 30($/MW day) of the 50-50 lottery is shown in Figure 7.2 and in the following equation:

$$\begin{aligned}\mu_{121}(30) &= 0.5\,\mu_{121}(1) + 0.5\,\mu_{121}(100)\\ &= 0.5 \; .\end{aligned}$$

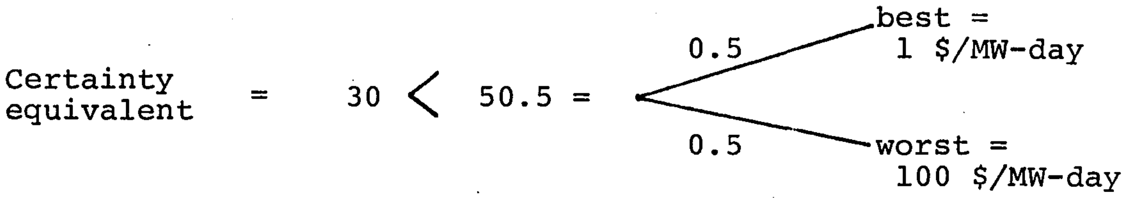


Figure 7.2. Certainty equivalent of the lottery for X_{121}

Since 30($/MW day) is less than expected value 50.5, this original assessment indicates the utility function might exhibit risk averse preference. In a similar way, a few more points can be chosen on the curve, e.g., 13 is indifferent to 1 and 30 ($/MW day); and 54 is indifferent to 30 and 100 ($/MW day), both for a 50-50 lottery. Thus,

$$\begin{aligned}\mu_{121}(13) &= 0.5\ \mu_{121}(1) + 0.5\ \mu_{121}(30)\\ &= 0.75\end{aligned}$$

and

$$\begin{aligned}\mu_{121}(54) &= 0.5\ \mu_{121}(30) + 0.5\ \mu_{121}(100)\\ &= 0.25\ .\end{aligned}$$

So far, we know five points corresponding to utility value 0, 0.25, 0.50, 0.75, and 1.0 which are sufficient to construct the curve of μ_{121} as shown in Figure 7.4(a).

This process is repeated for some categories, attributes, and each subattribute and the utility functions are fitted by exponential function of the form

$$\mu(x) = A + B \exp\ (Cx)\ . \tag{4}$$

Table 7.3. The coefficients of utility function for each subattribute

Symbol			Category/attribute/ subattribute	A	B	C
Z_1			Cost			
	Y_{11}		Capital cost			
		X_{111}	Reactor and building cost	1.046	-0.0406	0.065
		X_{112}	Supporting facility cost	1.042	-0.327	0.327
	Y_{12}		Running cost			
		X_{121}	Fuel cost	-0.179	1.202	-0.019
		X_{122}	Maintenance and operation cost	1.210	-0.180	0.762
		X_{123}	Staffing and management	1.147	-0.117	0.762
Z_2			Technological soundness			
	Y_{21}		Availability of material			
		X_{211}	Fuel	-0.0228	1.600	-0.0448
		X_{212}	Coolant	-0.386	0.386	0.013
		X_{213}	Moderator	-0.386	0.386	0.013
		X_{214}	Reflector	-0.386	0.386	0.013
	Y_{22}		Construction time	-0.096	2.015	-0.305
	Y_{23}		State-of-the-art-level	6.155	-6.115	-0.305
Z_3			Risk			
	Y_{31}		Reliability of operation			
		X_{311}	Equipment error	-0.002	2.000	-0.069
		X_{312}	Human error	-0.002	1.189	-0.173
		X_{313}	Scram insertion time	-0.162	1.338	-0.703
	Y_{32}		Radiation level			

Table 7.3. Continued

Symbol			Category/attribute/ subattribute	A	B	C
		X_{321}	Radiation on reactor top	-0.039	1.039	-0.131
		X_{322}	Radiation on general floor area	-0.039	1.039	-0.656
	Y_{33}		Compatibility with selected site	-0.198	0.198	-0.018
Z_4			Serviceability			
	Y_{41}		Training and teaching quality			
		X_{411}	Ease of operation	1.039	-0.277	-0.066
		X_{412}	Simplicity of fuel loading	1.783	-4.060	-0.016
		X_{413}	Fool-proof controllability and inherent safety	1.198	-7.254	-0.036
		X_{414}	Shutdown margin	567.0	-567.1	-0.036
	Y_{42}		Usefulness in basic research			
		X_{421}	Core accessibility	1.018	-15.19	-0.068
		X_{422}	Core flexibility	1.052	-3.822	-0.043
		X_{423}	Reactor power stability	546.2	-546.2	-0.00
		X_{424}	Ease of startup and shutdown	-0.096	1.096	-0.305
		X_{425}	Rapidity of power/ flux level change	1.120	-1.196	-0.066
	Y_{43}		Adaptability for applied research and development			
		X_{431}	Material testing research and development	1.00	-1.081	-0.770
		X_{432}	Life science research	1.039	-27.66	-0.066
		X_{433}	Science research	1.039	-27.66	-0.660

Table 7.3. Continued

Symbol			Category/attribute/ subattribute	A	B	C
	Y_{44}		Services salability			
		X_{441}	Radioisotope productivity	1.198	-7.254	-0.036
		X_{442}	Experimental volume availability	1.198	-7.254	-0.036
		X_{443}	Supporting facility	1.198	-7.254	-0.036
Z_5			Compatibility with Saudi Arabia			
	Y_{51}		Availability of required local resources			
		X_{511}	Local manpower	-1.275	1.275	0.017
		X_{512}	Ease of maintenance	1.001	-1014.0	-0.138
	Y_{52}		Ease of acquisition of imported resources			
		X_{521}	Foreign experts	-10920	10920	0.000
		X_{522}	Spare parts	-0.096	2.015	-0.122
		X_{523}	Generation of research reactor facility	1.007	-1.195	-0.086
	Y_{53}		Community acceptance			
		X_{531}	Educational people	1.783	-1.783	-0.008
		X_{532}	Public	3277	-3277	0.000
		X_{533}	Government	1.385	-1.385	-0.013
	Y_{54}		Future expandability	-1.030	0.655	0.011

The coefficients A, B, and C for each of them are found by the computer program and the results are given in Table 7.3. A computer fit is made for the utility functions and they are shown in Figures 7.3 through 7.18.

7.5. Assessment of Tradeoff Constants

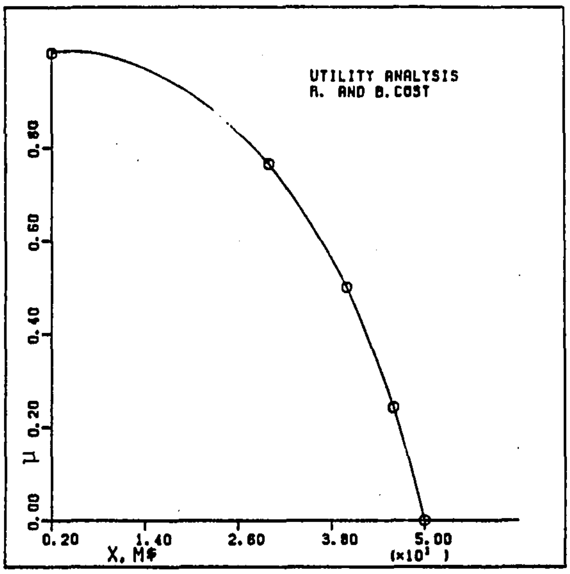
The important issue of tradeoffs among the categories, attributes in each category, and subattributes in each attribute is addressed by assessing the K_is. The assessment approach involves the following:

(1) ranking the constants to reflect the order of preference for the corresponding categories, attributes, or subattributes;

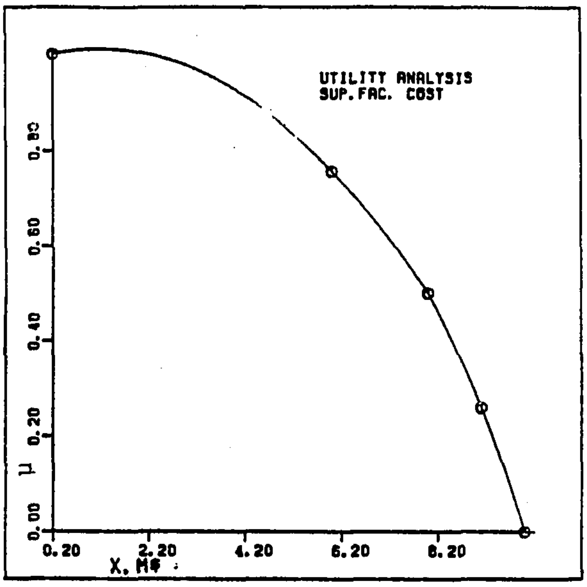
(2) establishing bilateral relationships among the constants for categories, attributes in each category, and subattributes in each attribute;

(3) finally, specifying their values.

To establish the ranking of the K_is, assume that all categories are at their worst levels, such as $(Z_1^0, Z_2^0, Z_3^0, Z_4^0, Z_5^0)$. Now, the first category which the author prefers to improve is Z_4, serviceability of the research reactor. This category is the most important one because it represents the main reason for establishing the nuclear research center and it examines the alternative facilities to choose the best one which meets the needs of the country. This implies that K_4 must be the largest constant to reflect the

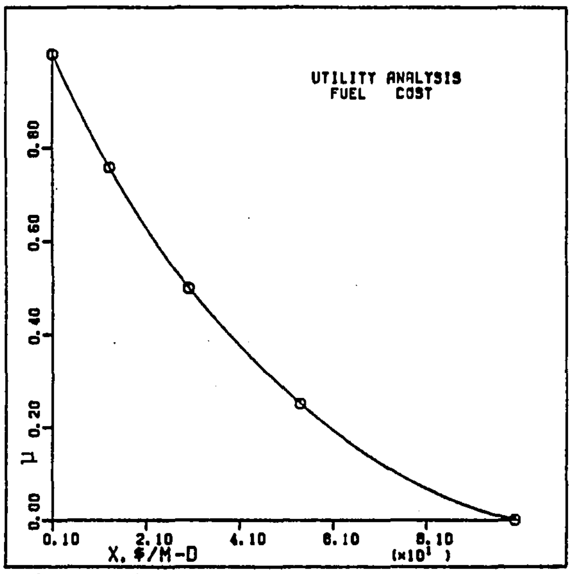


a) Reactor and building cost

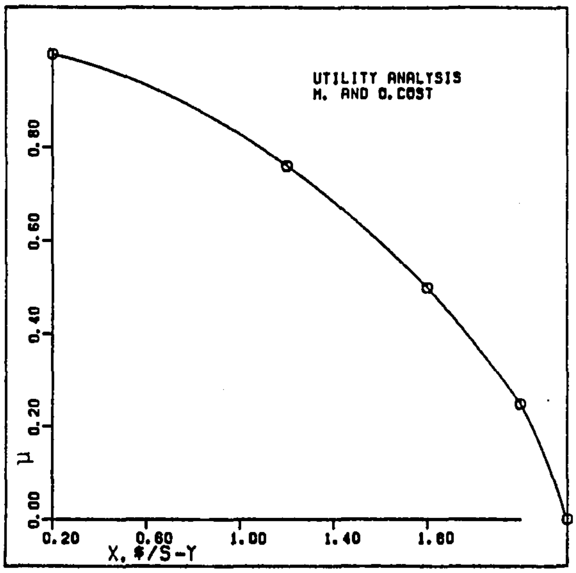


b) Support facilities cost

Figure 7.3. Utility functions for capital cost subattributes

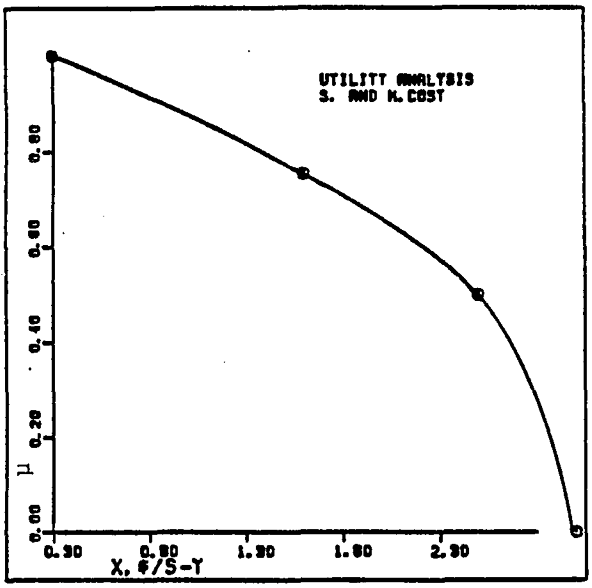


a) Fuel cost



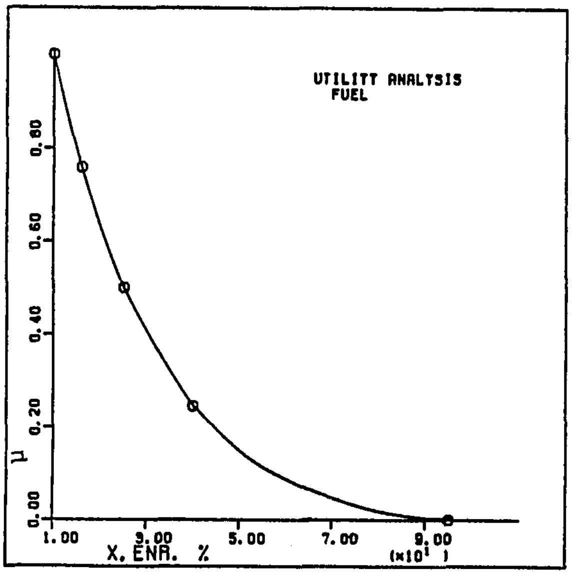
b) Maintenance and operation cost

Figure 7.4. Utility functions for operating cost subattributes

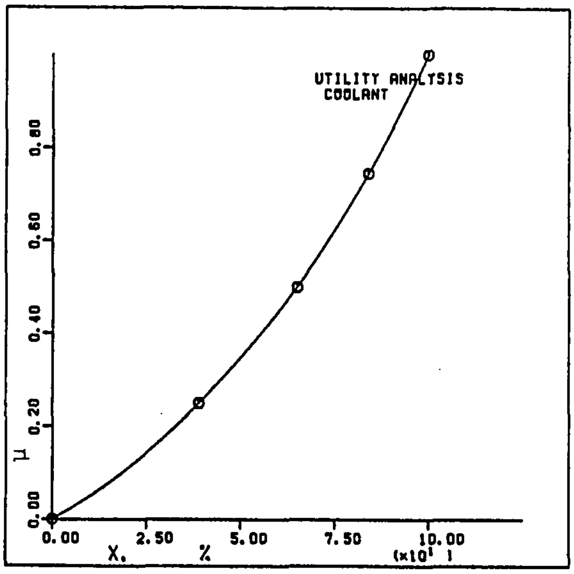


c) Staffing and management cost

Figure 7.4. (Continued)

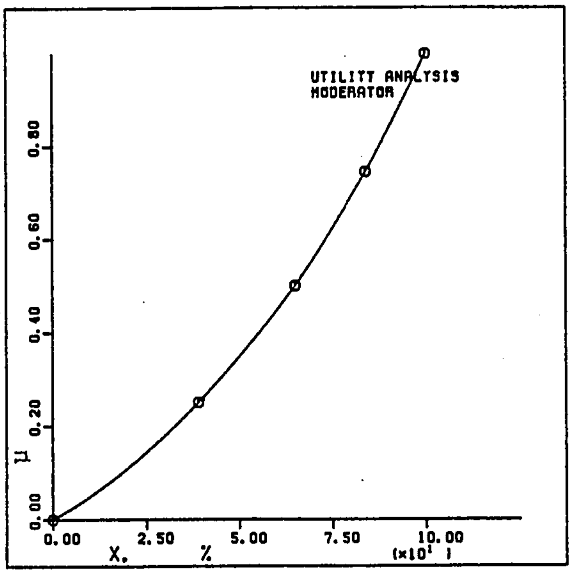


a) Availability of fuel

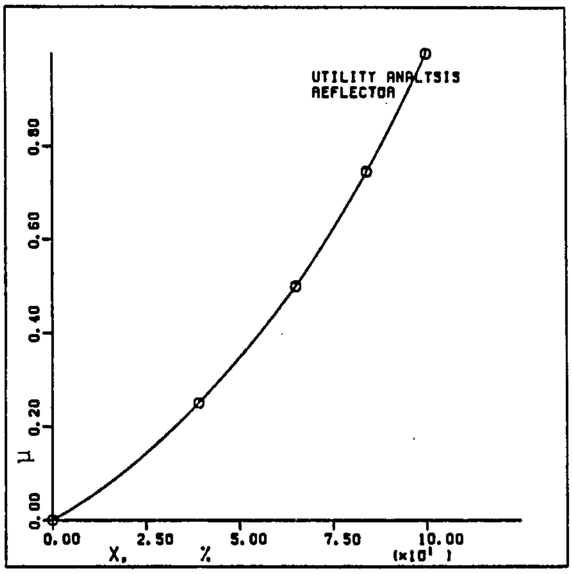


b) Availability of coolant

Figure 7.5. Utility functions for material availability subattributes



c) Availability of moderator



d) Availability of reflector

Figure 7.5. (Continued)

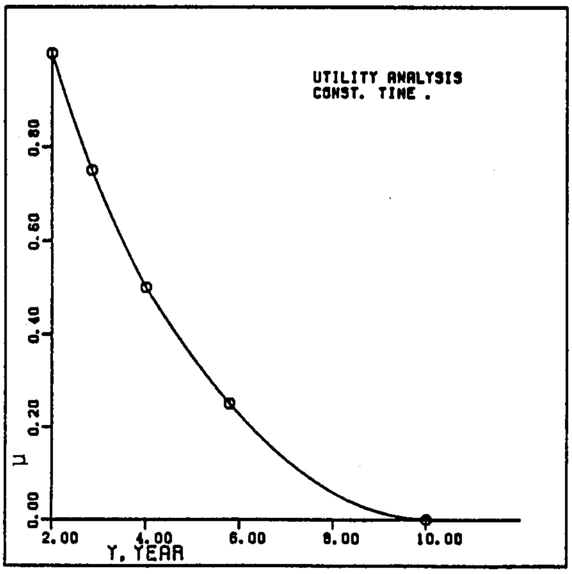


Figure 7.6. Utility function for construction time attribute

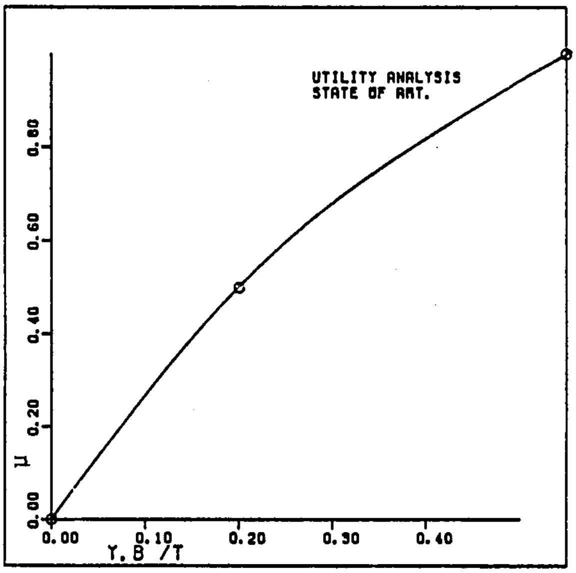
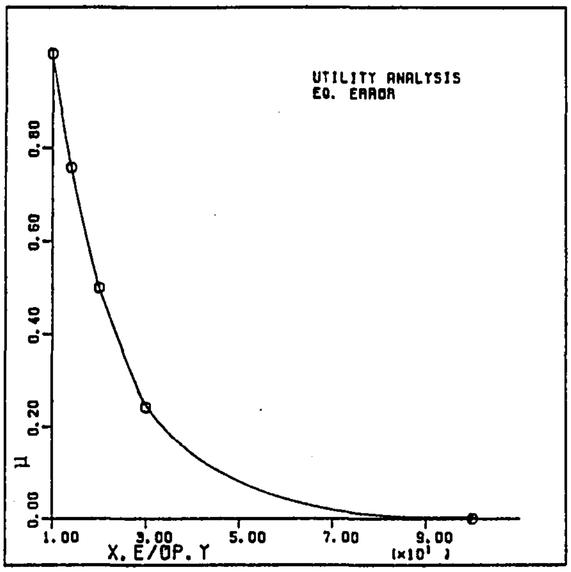
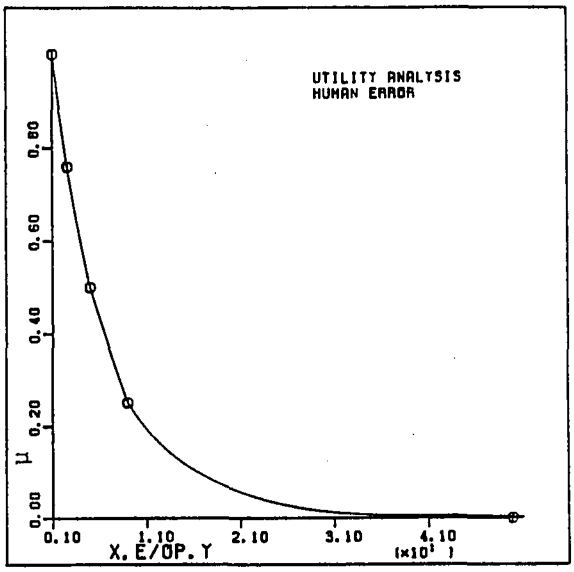


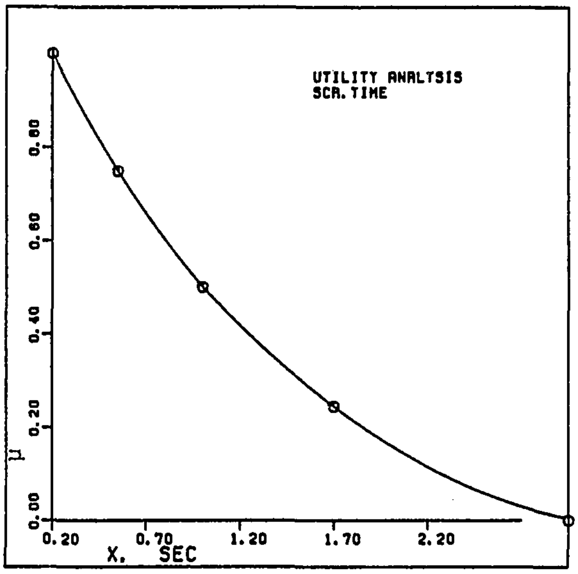
Figure 7.7. Utility function for state of the art attribute



a) Equipment failure

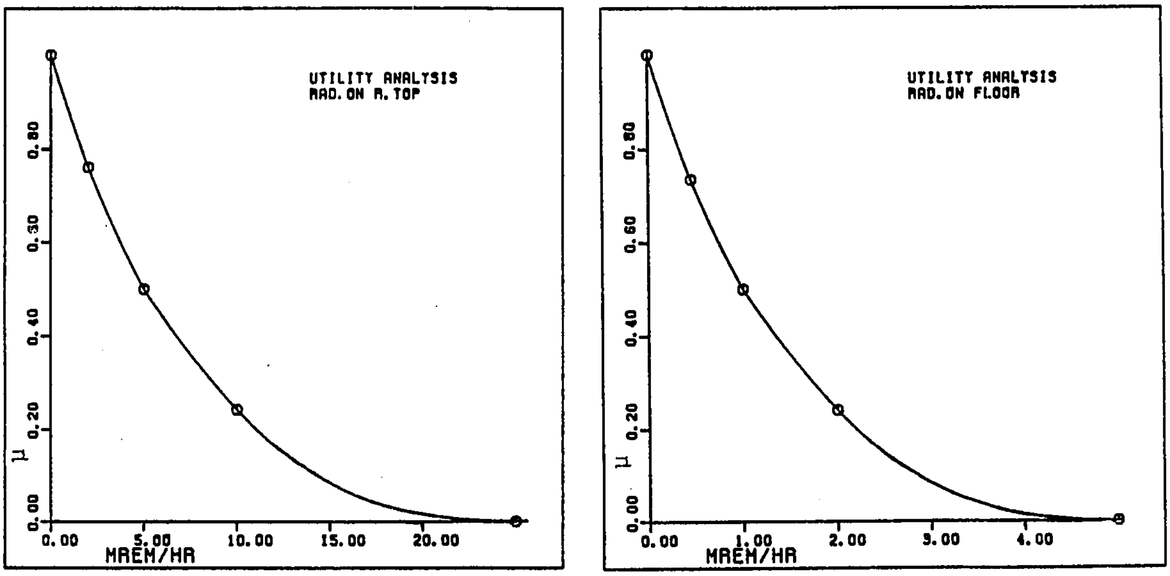


b) Human error



c) Scrame time

Figure 7.8. Utility functions for reliability of operation subattributes



a) Radiation level on reactor top

b) Radiation level on general floor area

Figure 7.9. Utility functions for radiation level subattributes

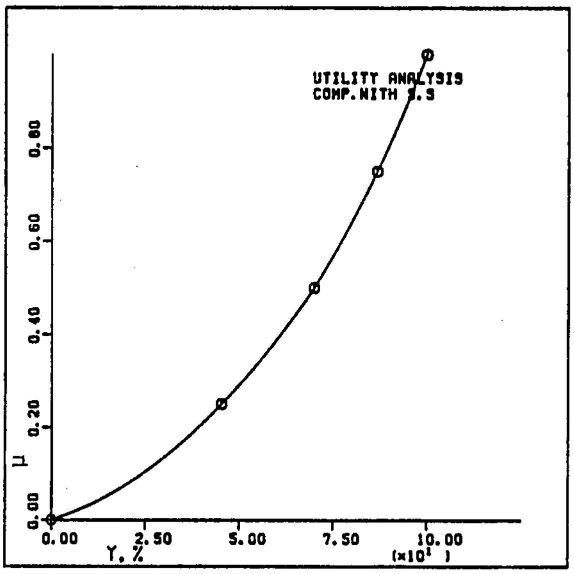
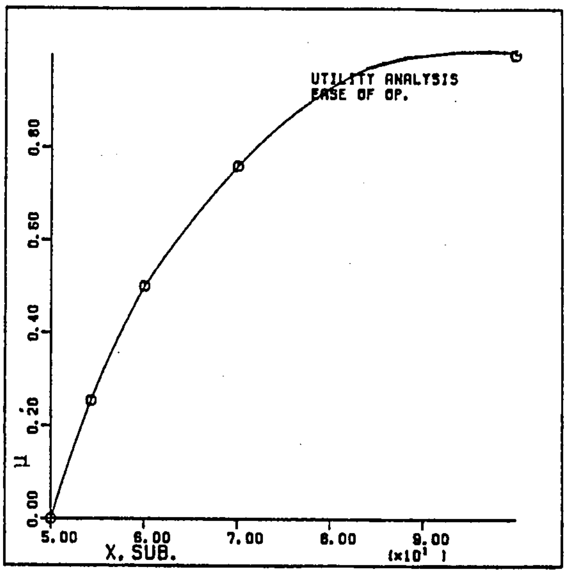
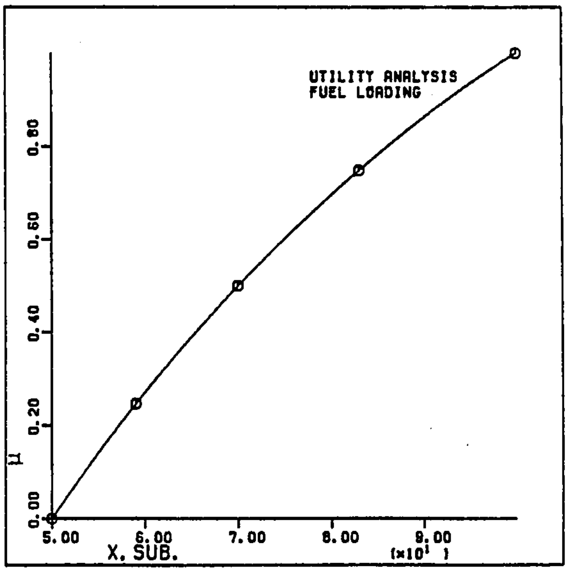


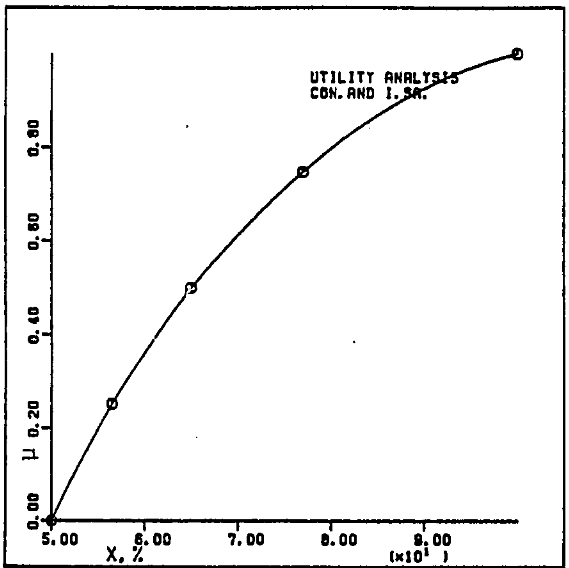
Figure 7.10. Utility function for site compatibility attribute



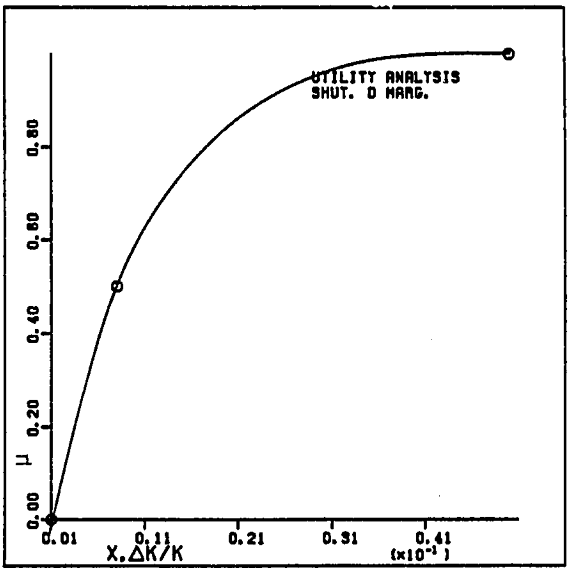
a) Ease of operation



b) Simplicity of fuel loading

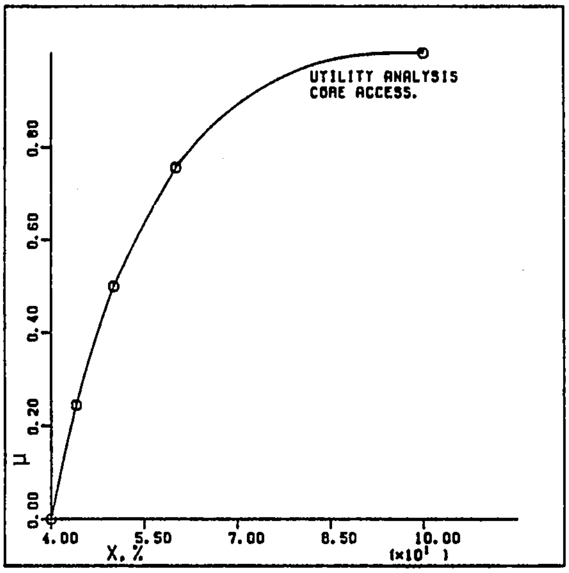


c) Foolproof controbility and inherent safety

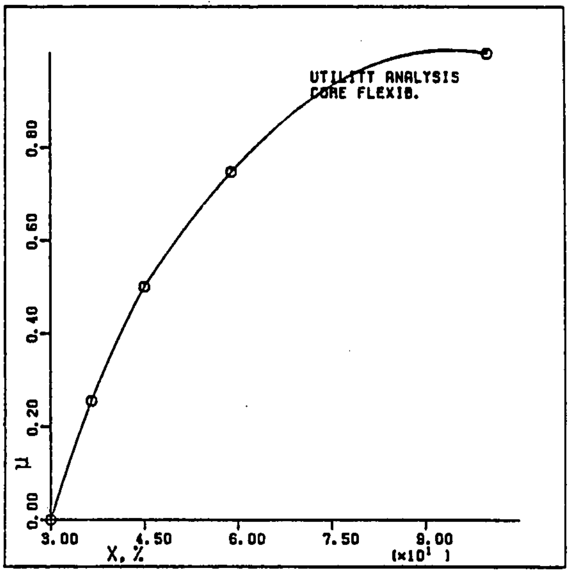


d) Shutdown margin

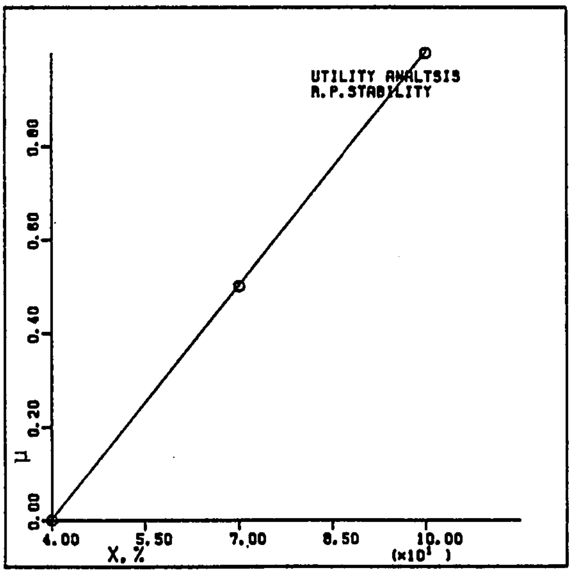
Figure 7.11. Utility functions for training and teaching quality subattributes



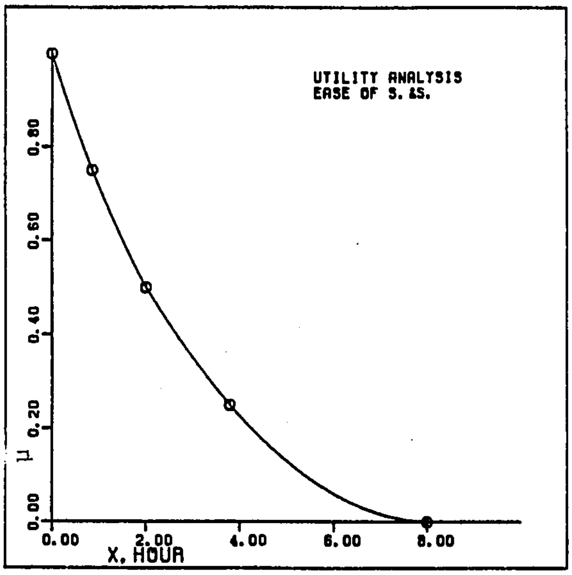
a) Core accessibility



b) Core flexibility

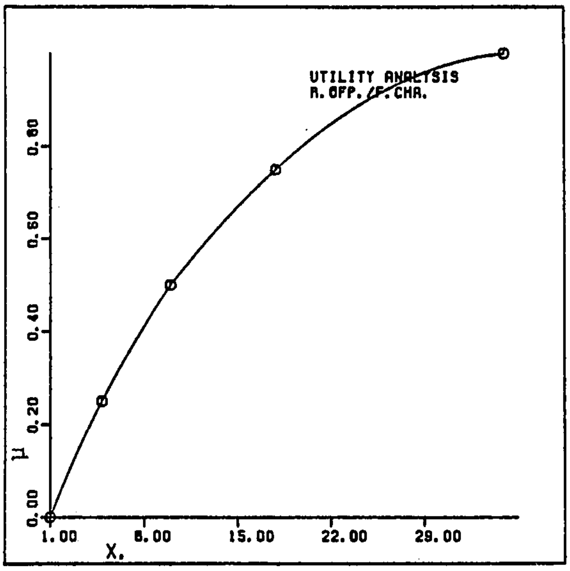


c) Reactor power stability



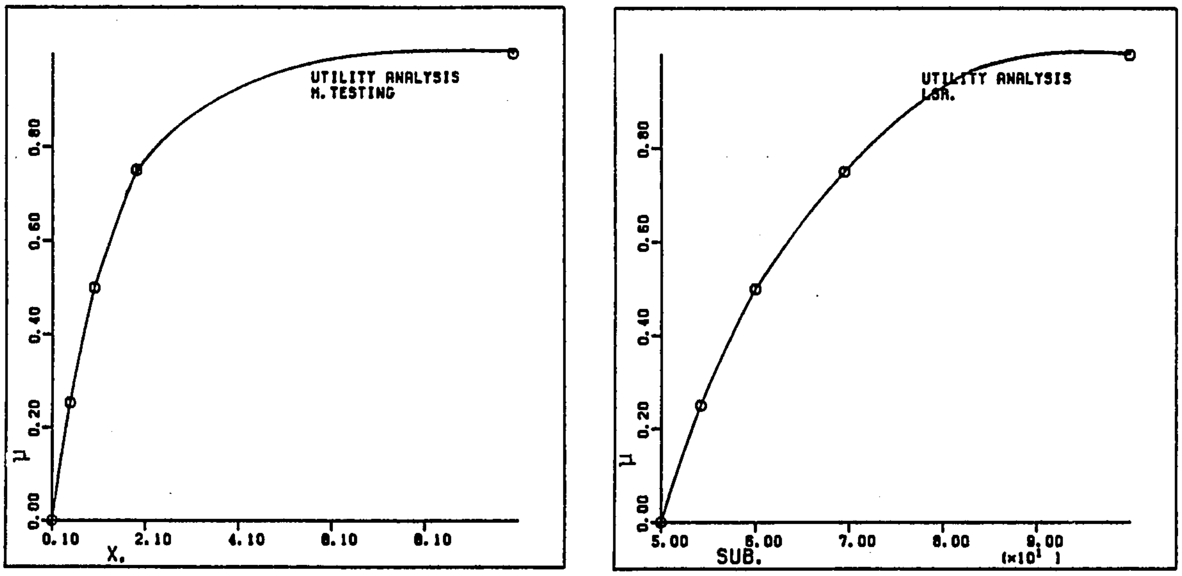
d) Ease of startup and shut-down

Figure 7.12. Utility functions for basic research sub-attributes



e) Rapidity of power/flux level change

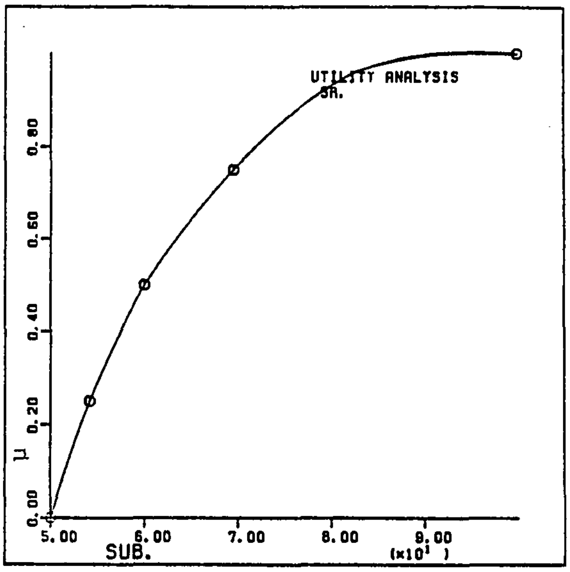
Figure 7.12. (Continued)



a) Material testing

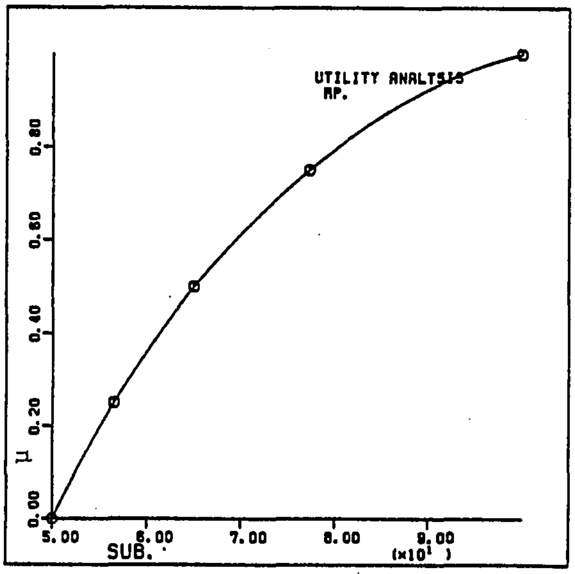
b) Life science research

Figure 7.13. Utility functions for applied research sub-attributes

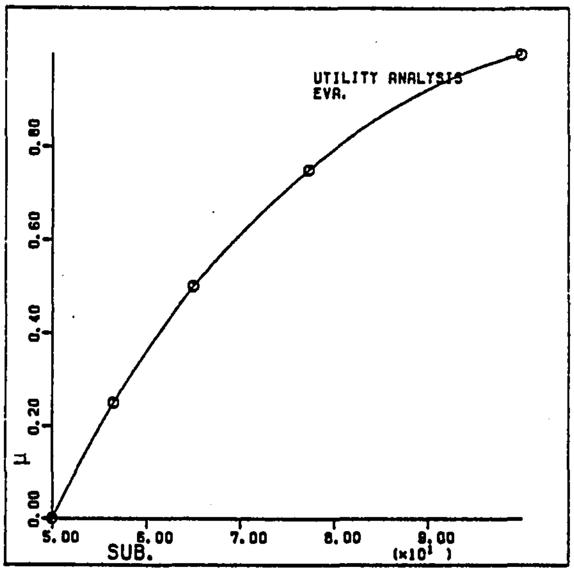


c) Science research

Figure 7.13. (Continued)

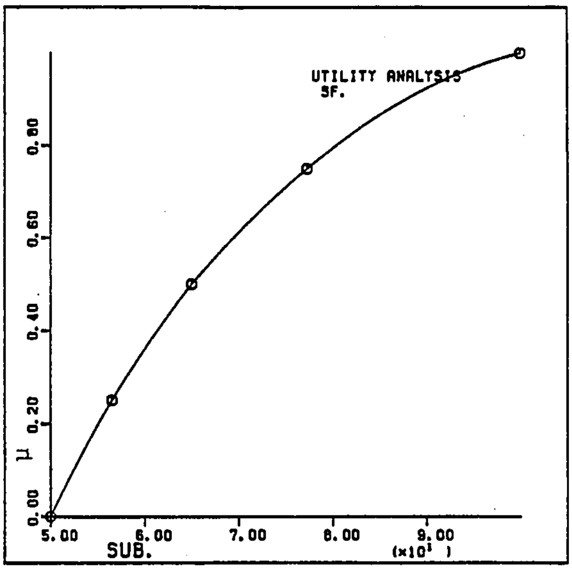


a) Radioisotope productivity



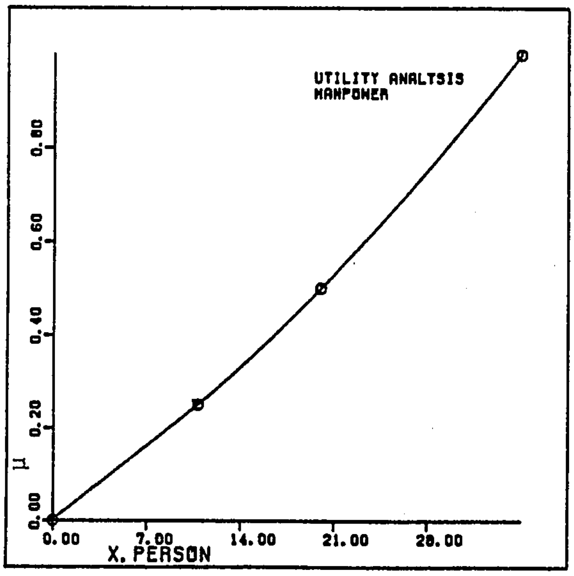
b) Experimental volume availability

Figure 7.14. Utility functions for salability subattributes

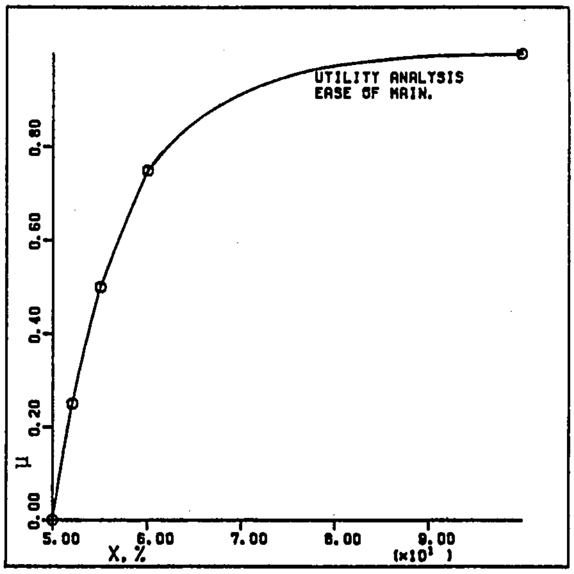


c) Supporting facilities

Figure 7.14. (Continued)

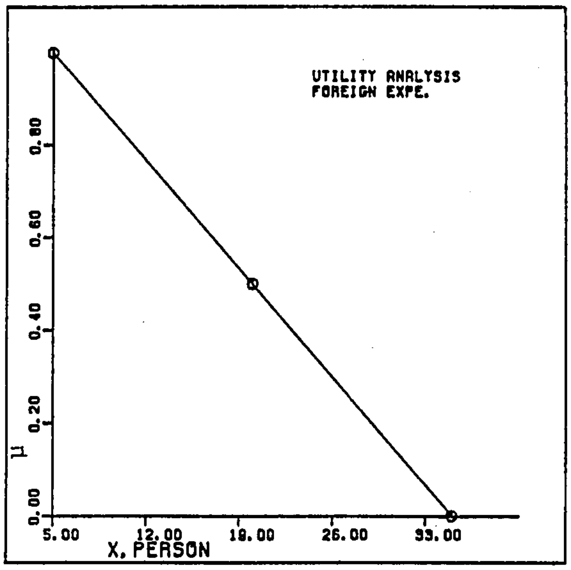


a) Manpower

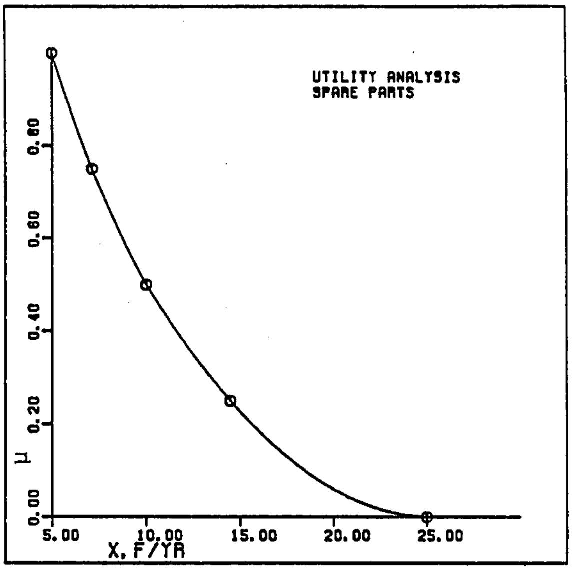


b) Ease of maintenance

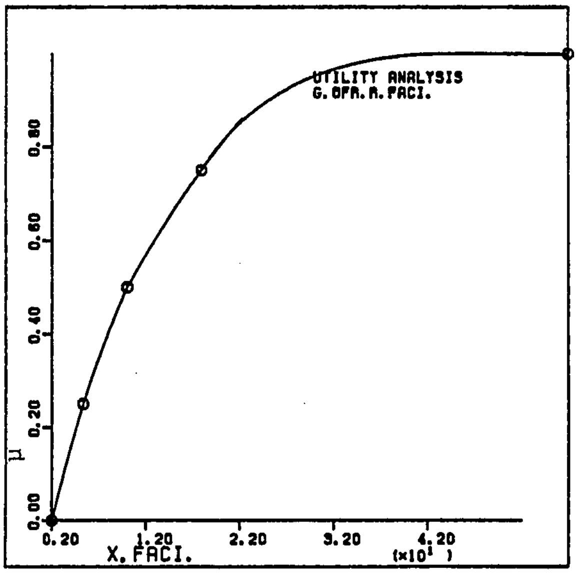
Figure 7.15. Utility functions for availability of local resources subattributes



a) Foreign experts

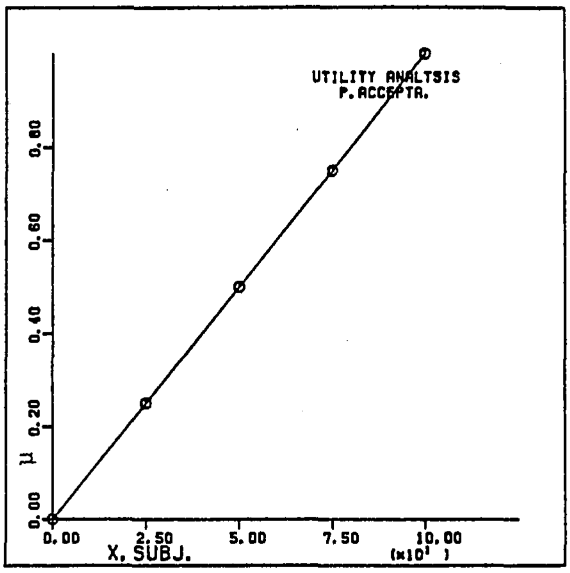


b) Spare parts

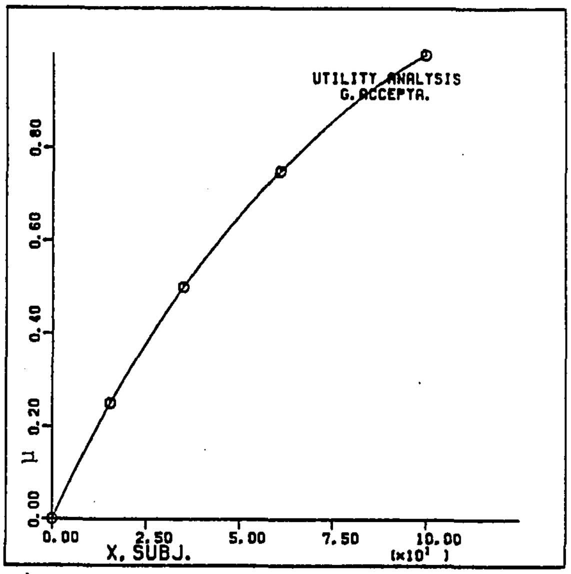


c) Generation of research reactor facilities

Figure 7.16. Utility functions for imported resources sub-attributes

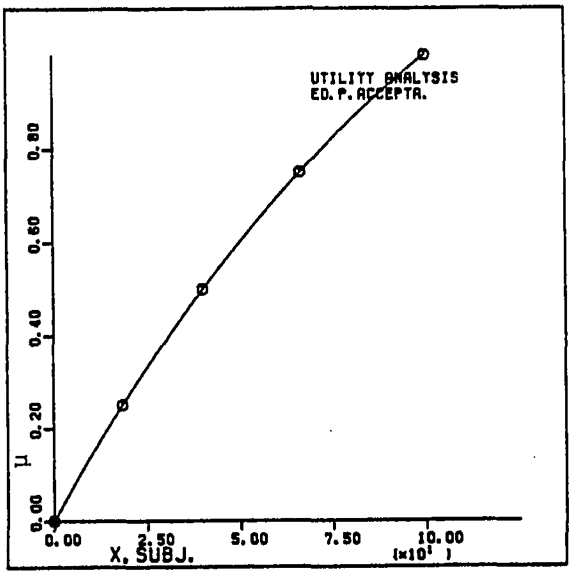


b) Public acceptance



c) Government acceptance

Figure 7.17. Utility functions for community acceptance sub-attributes



d) Educated people acceptance

Figure 7.17. (Continued)

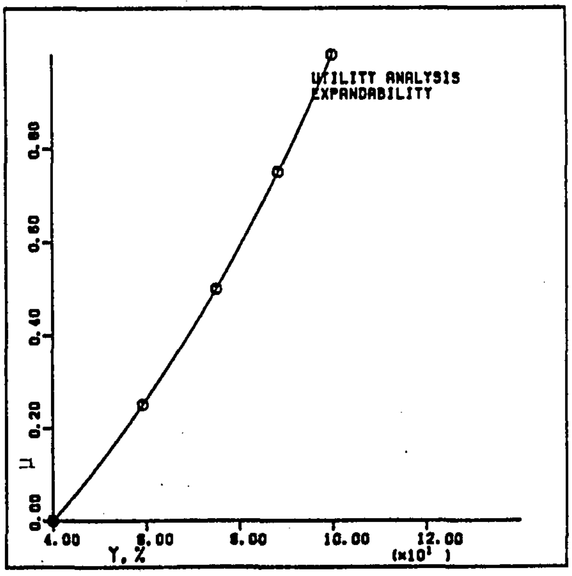


Figure 7.18. Utility function for future expandability attribute

importance of Z_4 on the composite utility scale. Then the process is repeated with the remaining categories until the complete ranking of the K_is is determined. From the author's perception, the following ranking is appropriate:

$$K_4 > K_5 = K_3 > K_2 > K_1 \quad .$$

The above choice of constant ranking is based on the following justifications. Category Z_4 represents the core of the project and that is to meet the various needs of the country. The constant K_5 ranks high because the project of establishing the nuclear center depends to a great extent on the society acceptance, especially the Saudi government. Although the government has made long range planning for introduction of nuclear technology, the recent political situation in the area has a great impact on starting the center. Also, K_3 ranks high because safety considerations should be taken seriously, especially during this time to help in nuclear transfer technology to the country easily and with minimum inconvenience. Usually, the first project of any type in the country creates good or bad images about this technology. The constant K_2 ranks about in the middle between K_3, safety considerations, and K_1, cost, because most research reactor types are proven types by successful operation from the standpoint of technological soundness. Also, there is no serious problems in the availability of the reactor materials in general. The constant K_1, cost, ranks low because the economic situation of Saudi Arabia differs greatly from other developing countries due to availability of financial resources. So this reflects that the government is willing to allocate any amount of funds for the project. The ranking order of attributes in each category is determined and also the ranking order of subattributes in each attribute is determined.

To directly face the tradeoff issue, the decision maker is asked to consider at the beginning the worst level of all categories and changing either Z_4 from Z_4^0 to some intermediate value $Z_4^{'}$ or Z_5 from Z_5^0 to Z_5^1 that is indifferent to the decision maker. Then, of course, the preference μ of the two indifferent consequences must be equal so that

$$\mu(Z_1^0,\ Z_2^0,\ Z_3^0,\ Z_4^{'},\ Z_5^0) = \mu(Z_1^0,\ Z_2^0,\ Z_3^0,\ Z_4^0,\ Z_5^1) \qquad (4)$$

but

$$\mu(Z_1, Z_2, \ldots, Z_5) = \sum_{i=1}^{5} K_i \mu_i (Z_i)$$

or

$$1 + K\mu(Z_1, \ldots, Z_5) = \sum_{i=1}^{5} [1 + KK_i \mu_i (Z_i)] \ .$$

Therefore, with the left side of equation 4 in the additive form

$$\begin{aligned} \mu(Z_1^0,\ Z_2^0,\ Z_3^0,\ Z_4^{'},\ Z_5^0) &= K_1\mu_1(Z_1^0) + K_2\mu_2(Z_2^0) + K_3\mu_3(Z_3^0) \\ &\quad + K_4\mu_4(Z_4^{'}) + K_5\mu_5(Z_5^0) \\ &= 0 + 0 + 0 + K_4\mu_4(Z_4^{'}) + 0 \\ &= K_4\mu_4(Z_4^{'}) \qquad . \qquad (5) \end{aligned}$$

In the multiplicative form

$$1 + K\mu(Z_1^0, Z_2^0, Z_3^0, Z_4^{'}, Z_5^0) =$$

$$[1 + KK_1\mu_1(Z_1^0)] \ldots [1 + KK_4\mu_4(Z_4^{'})][1 + K_5K\mu_5(Z_5^0)]$$

$$= 1 \times 1 \times 1 \times [1 + KK_4\mu_4(Z_4^{'})] \times 1$$

$$1 + K\mu(Z_1^0, Z_2^0, Z_3^0, Z_4^{'}, Z_5^0) = 1 + KK_4\mu_4(Z_4^{'})$$

$$\mu(Z_1^0, Z_2^0, Z_3^0, Z_4^{'}, Z_5^0) = K_4\mu_4(Z_4^{'}) \ . \qquad (6)$$

From equations 5 and 6, the result of the additive and multiplicative forms is equal.

Now with the right side of equation 4 in additive form

$$\mu(Z_1^0, Z_2^0, Z_3^0, Z_4^0, Z_5^1) = K_1\mu_1(Z_1^0) + K_2\mu_2(Z_2^0)$$

$$+ K_3\mu_3(Z_3^0) + K_4\mu_4(Z_4^0) + K_5\mu_5(Z_5^{'})$$

$$= 0 + 0 + 0 + 0 + K_5 \times 1$$

$$\mu(Z_1^0, Z_2^0, Z_3^0, Z_4^0, Z_5^1) = K_5 \ . \qquad (7)$$

In the multiplication form

$$1 + K\mu(Z_1^0, Z_2^0, Z_3^0, Z_4^0, Z_5^1) = [1 + KK_1\mu_1(Z_1^0)] \ldots [1 + KK_5\mu_5(Z_5^1)]$$

$$= 1 \times 1 \times 1 \times 1 \times [1 + KK_5\mu_5(Z_5^1)]$$

$$= 1 + KK_5$$

$$\mu(Z_1^0, Z_2^0, Z_3^0, Z_4^0, Z_5^1) = K_5 \ . \qquad (8)$$

From equations 7 and 8, also, the result of the additive and multiplicative form is equal. Then, from equations 5, 6 (the left side) and from equations 7 and 8 (the right side), then

$$K_5 = K_4\mu_4(Z_4^{'}) \ . \qquad (9)$$

But $K_5 = K_3 \rightarrow K_3 = K_4\mu_4(Z_4^{'})$. (10)

By a similar procedure, we get,

$$K_2 = K_4\mu_4(Z_4'') \tag{11}$$

and

$$K_1 = K_4\mu_4(Z_4''') \ . \tag{12}$$

Then, the procedure is repeated to find the tradeoffs between the attributes in each category and between the subattributes in each attribute. The resulting equations for categories, attributes, subattributes are listed in Table 7.4. The final values of K_is are calculated by the computer program and are also indicated in Table 7.4.

7.6. Calculation of the Multiattribute Utility Functions

Since the categories, attributes, and subattributes have satisfied the conditions of the multiattribute utility function as discussed earlier in this chapter, then the function is either multiplicative or additive. If the sum of K_is equals one, the additive form will be used as follows:

$$\mu(x_1,\ldots,x_i) = \sum_{i=1}^{n} K_i\mu_i(x_i)$$

$$\text{if} \quad \sum_{i=1}^{n} K_i = 1 \ .$$

If the sum of K_is does not equal one, the multiplicative form will be used as follows:

$$1 + K\mu(x_1,\ldots,x_n) = \prod_{i=1}^{n} [1 + KK_i\mu_i(x_i)]$$

$$\text{if} \quad \sum_{i=1}^{n} K_i \neq 1 \ .$$

Table 7.4. Ranks and tradeoff constants for categories, attributes and subattributes

Symbol	Category/attribute/ subattribute	Rank Z	Rank Y	Rank X	Tradeoff constant K_i
Z_1	Cost	4			$K_1 = K_4\mu_4(50) = 0.170$
Y_{11}	Capital cost		2		$K_{11} = K_{12}\mu_{12}(30) = 0.406$
X_{111}	Reactor and building cost			1	$K_{111} = 0.500$
X_{112}	Supporting facility cost			2	$K_{112} = K_{111}\mu_{11}(10) = 0.484$
Y_{12}	Running cost		1		$K_{12} = 0.550$
X_{121}	Fuel cost			3	$K_{121} = K_{123}\mu 123(1) = 0.313$
X_{122}	Maintenance and operation cost			2	$K_{122} = K_{123}\mu_{123}(.5) = 0.342$
X_{123}	Staffing and management cost			1	$K_{123} = 0.350$
Z_2	Technological soundness	3			$K_2 = K_4\mu_4(75) = 0.202$
Y_{21}	Availability of material		1		$K_{21} = 0.450$
X_{211}	Fuel			1	$K_{211} = 0.700$
X_{212}	Coolant			2	$K_{212} = K_{211}\mu_{211}(50) = 0.104$
X_{213}	Moderator			2	$K_{213} = K_{211}\mu_{211}(50) = 0.104$
X_{214}	Reflector			2	$K_{214} = K_{211}\mu_{211}(50) = 0.104$
Y_{22}	Construction time		3		$K_{22} = K_{21}\mu_{21}(80) = 0.251$
Y_{23}	State-of-the-art level		2		$K_{23} = K_{21}\mu_{21}(80) = 0.309$

Table 7.4. Continued

Symbol			Category/attribute/ subattribute	Rank Z	Rank Y	Rank X	Tradeoff constant K_i
Z_3			Risk	2			$K_3 = K_4\mu_4(85) =$
	Y_{31}		Reliability of operation		1		$K_{31} = 0.55$
		X_{311}	Equipment error			2	$K_{311} = K_{312}\mu_{312}(4) = 0.327$
		X_{312}	Human error			1	$K_{312} = 0.550$
		X_{313}	Scram insertion time			3	$K_{313} = K_{312}\mu_{312}(25) = 0.010$
	Y_{32}		Radiation level		2		$K_{32} = K_{31}\mu_{31}(50) = 0.533$
		X_{321}	Radiation on reactor top			1	$K_{321} = 0.500$
		X_{322}	Radiation on general floor area			1	$K_{322} = 0.500$
	Y_{33}		Compatibility with selected site		3		$K_{33} = K_{31}\mu_{31}(25) = 0.453$
Z_4			Serviceability	1			$K_4 = 0.22$
	Y_{41}		Training and teaching quality		3		$K_{41} = K_{42}\mu_{42}(75) = 0.249$
		X_{411}	Ease of operation			1	$K_{411} = 0.250$
		X_{412}	Simplicity of fuel loading			4	$K_{412} = K_{411}\mu_{411}(75) = 0.209$
		X_{413}	Foolproof controllability and inherent safety			3	$K_{413} = K_{411}\mu_{411}(80) = 0.224$
		X_{414}	Shutdown margin			2	$K_{414} = K_{411}\mu_{411}(90) = 0.241$

Table 7.4. Continued

Symbol	Category/attribute/subattribute	Rank Z	Rank Y	Rank X	Tradeoff constant K_i
Y_{42}	Usefulness in basic research		1		$K_{42} = 0.28$
X_{421}	Core accessibility			1	$K_{421} = 0.20$
X_{422}	Core flexibility			3	$K_{422} = K_{421}\mu_{421}(85) = 0.194$
X_{423}	Reactor power stability			2	$K_{423} = K_{421}\mu_{421}(95) = 0.199$
X_{424}	Ease of startup and shutdown			5	$K_{424} = K_{421}\mu_{421}(70) = 0.177$
X_{425}	Rapidity of power/flux level change			4	$K_{425} = K_{421}\mu_{421}(80) = 0.189$
Y_{43}	Adaptability for applied research and development		2		$K_{43} = K_{42}\mu_{42}(90) = 0.269$
X_{431}	Material testing research			3	$K_{431} = K_{432}\mu_{432}(70) = 0.243$
X_{432}	Life science research			1	$K_{432} = 0.32$
X_{433}	Science research			2	$K_{433} = K_{432}\mu_{432}(80) = 0.286$
Y_{44}	Services salability		3		$K_{44} = K_{42}\mu_{42}(75) = 0.249$
X_{441}	Radioisotope productivity			1	$K_{441} = 0.35$
K_{442}	Experimental volume availability			3	$K_{442} = K_{411}\mu_{411}(70) = 0.215$
K_{443}	Supporting facility			2	$K_{443} = K_{411}\mu_{411}(80) = 0.277$

Table 7.4. Continued

Symbol	Category/attribute/ subattribute	Rank Z	Rank Y	Rank X	Tradeoff constant K_i
Z_5	Compatibility with Saudi Arabia	2			$K_5 = K_4 \mu_4 (85) = 0.211$
Y_{51}	Availability of required local resources		2		$K_{51} = K_{53} \mu_{53} (80) = 0.240$
X_{511}	Local manpower			1	$K_{511} = 0.55$
X_{512}	Ease of maintenance			2	$K_{512} = K_{511} \mu_{511} (25) = 0.359$
Y_{52}	Ease of acquisition of imported resources		3		$K_{52} = K_{53} \mu_{53} (75) = 0.231$
X_{521}	Foreign experts			1	$K_{521} = 0.50$
X_{522}	Spare parts			3	$K_{522} = K_{521} \mu_{521} (30) = 0.083$
X_{523}	Generation of research facility			2	$K_{533} = K_{521} \mu_{521} (25) = 0.167$
Y_{53}	Community acceptance		1		$K_{53} = 0.27$
X_{531}	Educated people			2	$K_{531} = K_{533} \mu_{533} (70) = 0.475$
X_{532}	Public			3	$K_{532} = K_{533} \mu_{533} (40) = 0.322$
X_{533}	Government			1	$K_{533} = 0.58$
Y_{54}	Future expandability		4		$K_{54} = K_{53} \mu_{53} (60) = 0.201$

The value of the parameter K in the above equation can be determined by evaluating it at $(x_1^1, x_2^1, \ldots, x_n^1)$ where x_i^1 is the best level of x_i. This gives

$$1 + K\mu(x_1^1, x_2^1, \ldots, x_n^1) = \prod_{i=1}^{n} [1 + KK_i\mu_i(x_i^1)]$$

But

$$\mu(x_1^1, x_2^1, \ldots, x_n^1) = 1$$

and

$$\mu_i(x_i^1) = 1 \ .$$

Hence,

$$1 + K = \prod_{i=1}^{n} (1 + KK_i) \quad .$$

By the developed computer program, the two forms can be calculated. In the case of the multiplicative form, K will be obtained by the same computer program using iterative procedure (see Appendix B).

The utility function for each alternative was calculated using the additive or multiplicative form by the computer program given in Appendix B. The value of the utility for each of the alternative facilities are given in Table 7.5. The most desirable facility is the one that corresponds to the highest utility value. Thus, we note that FNR facility has higher overall utility than the MITR. WUNR facility has highest overall utility while the GTRR facility has the lowest overall utility.

Table 7.5. Utility value for alternative facilities

	Category	FNR	MITR	GTRR	UWNR
Z_1	Cost	0.878	0.642	0.865	0.973
Rank		2	4	3	1
Z_2	Technological soundness	0.395	0.383	0.205	0.697
Rank		2	3	4	1
Z_3	Safety	0.630	0.813	0.482	0.555
Rank		2	1	4	3
Z_4	Serviceability	0.818	0.751	0.750	0.693
Rank		1	2	3	4
Z_5	Compatibility with Saudi Arabia	0.591	0.424	0.349	0.652
Rank		2	3	4	1
	Overall utility	0.667	0.613	0.529	0.714
Rank		2	3	4	1

8. SITING

8.1. Introduction

Siting of nuclear research center is a complex problem since it impacts population safety [91-98]. There are several interrelated factors influencing site selection which make site selection a multidimensional decision problem and, hence, multiattribute decision theories may be used.

One approach is to use the multiattribute utility theory which has been outlined by Gross et al. [93] and has been applied to specific sites in Kansas by Ahmed et al. [91]. The approach allows for including subjective site attributes by using arbitrary quantitative measures. Among the intangibles considered is public preference [91, 99]. Simpler rating and ranking techniques are also used [92, 100, 101]. However, assignment of numerical values to various attributes lacks precision and sensitivity analysis becomes necessary since decisions may be reversed if numerical values are changed for attributes of large weights. The use of verbal judgment can overcome this difficulty since each rating or weight is represented by a verb which is described by a fuzzy set. Fuzzy set theory has been used in decision analysis in various applications [50, 102-103]. Fuzzy decision applications to site selection have been first introduced by Kenarangui et al. [98]. The approach has been developed further here and applied to the specific case of selection of a site for the first nuclear research center in Saudi Arabia. Two specific locations are considered, one on the East Coast near Dhahran and the second near the city of Jeddah on the West Coast. The two locations are in a region where very good educational institutions are surrounded and various types of facilities are available near both sites which are not found in other regions of the country.

Comparisons have been made between the application of fuzzy decision analysis and other decision-making methodologies [50, 104], especially the multiattribute utility (MAU) theory. In principle, fuzzy sets can be used in conjunction with the MAU approach wherein values can be exchanged by fuzzy sets. The major advantage in this situation is to save the decision-maker the labor of coming up with exact numerical values and assigning specific uncertainties to such values. Hence, the fuzziness associated with each set will account for inherent imprecisions in value judgment. The approach employed here is a straightforward employment of the weighted rates methods to rank a set of given alternatives. Although a fuzzy MAU technique could be developed, the simpler ranking method used here is satisfactory for demonstrating the fuzzy set approach, although the fuzzy decision theory is highly mathematical and requires extensive exposure to fuzzy sets. However, the application of the theory to engineering decisions such as siting problems does not require any such labor beyond the use of the approach without or with little knowledge of the concept. The application of the method can be further facilitated by the use of interactive computer modules. In this situation, the decision process would simply involve verbal rating and weighing of attributes by the decision-makers directly into the computer or through programmers. Computer programs such as MAFDA [105] which is employed here can be readily used or other programs can be developed for special purposes by decision analysts.

8.2. Selection Principles

Earlier work on siting [91-101] has been reviewed to select the necessary criteria and safety requirements for selection of sites for nuclear research center in Saudi Arabia. Table 8.1 lists the criteria (attributes or merits) and subcriteria (subattributes) recommended for selection of sites of nuclear research center in Saudi Arabia. These criteria include topographical, oceanographical, geological, hydrological, seismological, and meteorological conditions. Also included is the distribution of population centers and availability of transportation, cooling water, construction facilities, support services, and domestic water.

Table 8.1. List of criteria recommended for siting of nuclear research center in Saudi Arabia

Symbol	Criteria	Subcriteria
X_1	Topography and oceanography	
Y_{11}		Ground level and easy access
Y_{12}		Site surveys
Y_{13}		Water depth
Y_{14}		Tides and tidal currents
X_2	Geology, hydrology and seismology	
Y_{21}		Geological structure
Y_{22}		Soil characteristics
Y_{23}		Underground water table
Y_{24}		Water sources
Y_{25}		Earthquake
Y_{26}		Seismic instruments
X_3	Meteorology	
Y_{31}		Rainfall
Y_{32}		Thunderstorm
Y_{33}		Humidity
Y_{34}		Fogs and mists
Y_{35}		Dust and sandstorms
Y_{36}		Temperature
Y_{37}		Wind
X_4	Transportation	
Y_{41}		Proximity of the site to main highway
Y_{42}		Capability of roads
X_5	Population	
Y_{51}		Density

Table 8.1. Continued

Symbol	Criteria	Subcriteria
y_{52}		Seasonal variation
X_6	Cooling water	
y_{61}		Sources
y_{62}		Availability
y_{63}		Distance
y_{64}		Characteristics
X_7	Construction, services, and domestic water	
y_{71}		Land properties for construction
y_{72}		Labor for construction
y_{73}		Construction difficulties
y_{74}		Electricity
y_{75}		Gas
y_{76}		Drainage and sewerage
y_{77}		Transmission and distribution lines
y_{78}		Availability of domestic water
y_{79}		Characteristics of domestic water
y_{710}		Distribution systems for domestic water

8.2.1 Topography and oceanography

The study of topography of Jeddah and Dhahran regions shows that some of the Jeddah sites are in or near alleys (Wadis) at their outlets to the coastal plain. From a topographical point of view, this represents a risk of flooding in the Wadis. Some of the Jeddah sites are extremely flat and are in protected zones of the Wadis. The Dhahran region may constitute a more suitable site for nuclear centers with respect to this criterion because there is no trace of any Wadis.

From an oceanographical point of view, the study of the tides and tsunamis in both the Red Sea and Arabian Gulf concluded that the risk appears to be larger for Jeddah than for Dhahran.

8.2.2 Geology, hydrology, and seismology

The two regions of interest may be distinguished from the general geology of Arabia. In the West, there is the Precambrian basement, where the coastal zones are covered by Cenozoic sediments on which Jeddah is situated. In the East, there is a sedimentary basin proper, with the Rub Al Khali depression, wherein very thick Tertiary sediments are covered by Quaternary deposits on which Dhahran is situated. The detailed geological study of the two zones shows that both regions have satisfactory geology.

The specific hydrological problems for construction of a nuclear plant are constituted by the water supply, the pollution risks to the underground aquifers, and by polluting discharges. Eastern and Western regions are both coastal. Regarding waste disposal, the Dhahran region seems in danger of a possible seepage risk, but there is no specific risk for the Jeddah region. As for water supply of both regions, it seems advisable to use seawater desalination systems.

Seismic risks are considered a natural hazard which is difficult to control. Fortunately, Saudi Arabia is not seismically active. It can be concluded that a slight influence of Zagros seismicity exists on the Eastern region near Dhahran without any significant activity. On the other hand, a possible seismic hazard exists in the Western region near Jeddah.

8.2.3 Meteorology

The most important of meteorological conditions is the wind and next in importance is the temperature variation with the altitude. Rainfall appears to be more concentrated in Jeddah than in Dhahran. Relative humidity is consistently high in Jeddah. Sand and dust storms are unusual in Jeddah, but they are frequent in Dhahran. Temperature variation is smaller in Jeddah.

8.2.4 Transportation

The road network is well-developed in both regions. There are two ports on the West coast, Jeddah and Yanbo, where roll-on-roll-off goods transport is possible. In addition, air freight service is more developed in Jeddah than in Dhahran.

8.2.5 Population

The importance of the pilgrimage to the holy places, mainly Mecca and Medina, which are near Jeddah, should be emphasized. It is a once-a-year occurrence and in a limited length of time. From a population density point of view, the two regions are suitable, but for the Western region the influx of pilgrims (about 2 million yearly) should be taken into consideration in determining a particular site location.

8.2.6 Cooling water

Water resources of Saudi Arabia display great variation in quality, quantity, and depth. They depend very much on the geological constitution of the land. Both the Dhahran region and the Jeddah region have a similar situation concerning shortage in water supply and scarcity of ground water. The problem may be solved by desalination of the water from either the Red Sea or the Arabian Gulf.

8.2.7 Construction, services and domestic water

Both Dhahran and Jeddah regions have similar conditions in regard to suitability for construction, and availability of support services and domestic water. They have rapid development which is accompanied by continued industrialization and vast change in every aspect of services and social life.

8.3. Analysis

Fuzzy sets theory is used here to compare the sites near Jeddah and Dhahran to determine which site is the most preferable. This involves evaluation of three main parameters:

(1) Criteria weights

(2) Rating

(3) Ranking and preferability.

8.3.1 Criteria weights

Since some of the criteria are more important than the others, different weights should be assigned to different criteria to illustrate the difference of the importance of each criterion. This is done also to the subcriteria. To indicate the relative importance of differences between criteria, verbal weights are used. These weights may be represented by the membership function shown in Figure 8.1, or the fuzzy set, W_j:

$$W_j = \{(W_j),\ \mu_{W_j}\ (W_j)\},\ j=1,2,\ldots,n$$

$$W_j = \text{very important} = \{(0.8,0.0),\ (0.875,0.5),\ (0.95,1.0),\ (0.975,0.5),\ (1.0,0.0)\}$$

$$W_j = \text{important} = \{(0.6,0),\ (0.7,0.5),\ (0.8,1),\ (0.9,0.5),\ (1.0,0.0)\}$$

$$W_j = \text{moderately important} = \{(0.3,0.0),\ (0.4,0.5),\ (0.5,1.0),\ (0.6,0.5),\ (0.7,0.0)\}$$

$$W_j = \text{rather unimportant} = \{(0.0,0.0),\ (0.1,0.5),\ (0.2,1),\ (0.3,0.5),\ (0.4,0.0)\}$$

$$W_j = \text{very unimportant} = \{(0.0,0.0),\ (0.025,0.5),\ (0.05,1.0),\ (0.125,0.5),\ (0.25,0.0)\}\ .$$

The weights for the two alternatives, Jeddah and Dhahran, are shown in Table 8.2.

8.3.2 <u>Rating</u>

Each alternative is rated verbally with respect to each criterion alone, independently of all other alternatives. The ratings may be represented by the membership function shown in Figure 2, or by the fuzzy rating of an alternative

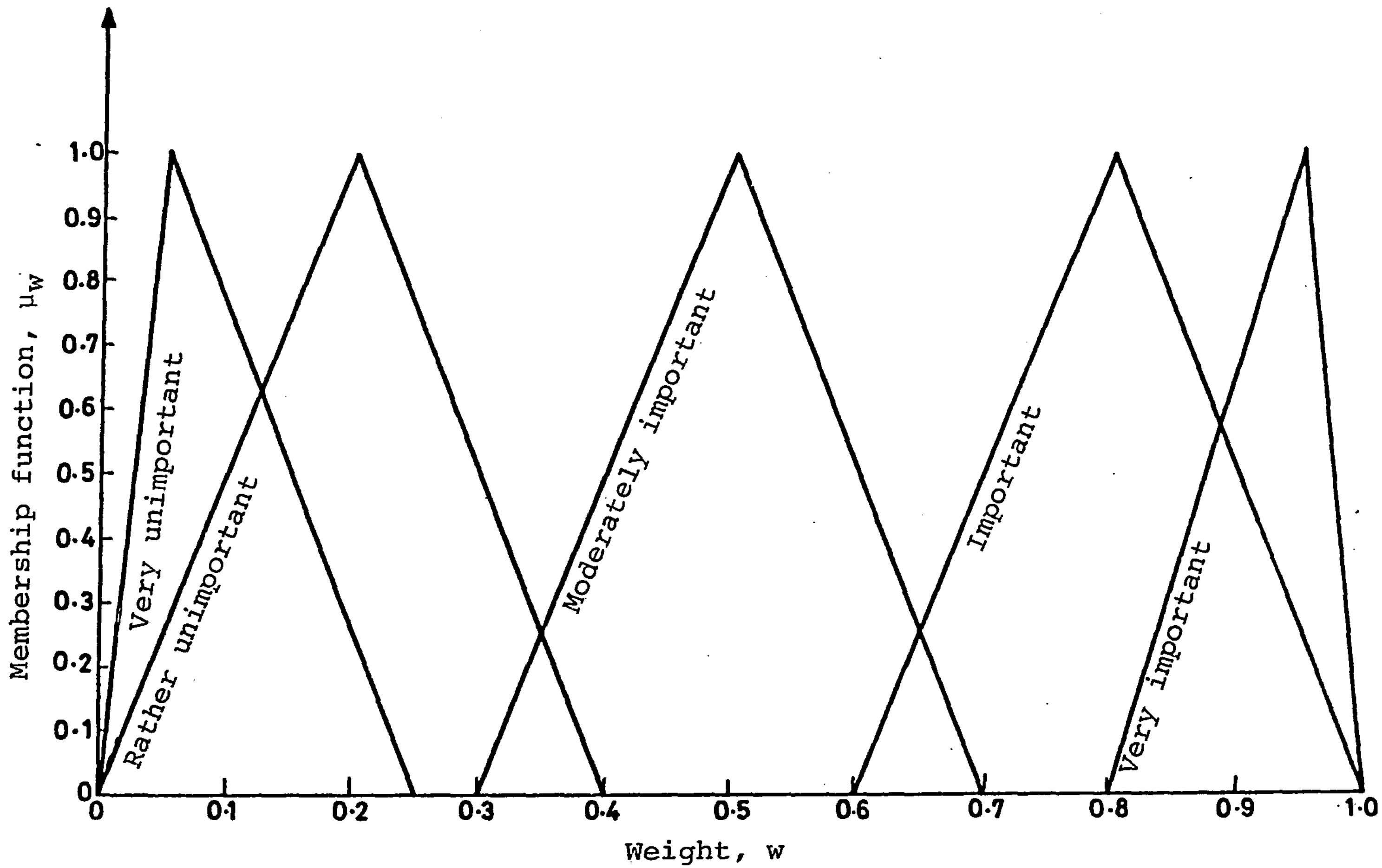


Figure 8.1. Membership function μ_w vs. weight w

Table 8.2. Weighting and rating for the two sites recommended for a nuclear research center in Saudi Arabia

Criteria	Subcriteria	Weights	Alternatives: Jeddah(1)	Alternatives: Dhahran(2)
X_1		Moderately important		
	y_{11}	Very important	Fair	Very good
	y_{12}	Moderately important	Fair	Very good
	y_{13}	Important	Fair	Good
	y_{14}	Moderately important	Fair	Good
X_2		Important		
	y_{21}	Important	Good	Good
	y_{22}	Important	Good	Good
	y_{23}	Very important	Good	Good
	y_{24}	Important	Good	Good
	y_{25}	Very important	Fair	Very good
	y_{26}	Moderately important	Fair	Fair
X_3		Important		
	y_{31}	Very unimportant	Good	Very good
	y_{32}	Very unimportant	Good	Fair
	y_{33}	Very unimportant	Fair	Good
	y_{34}	Rather unimportant	Good	Fair
	y_{35}	Moderately important	Good	Poor
	y_{36}	Important	Fair	Fair
	y_{37}	Very important	Very good	Fair
X_4		Important		
	y_{41}	Important	Fair	Fair
	y_{42}	Very important	Very good	Very good
X_5		Very important		
	y_{51}	Important	Good	Good
	y_{52}	Very important	Fair	Very good

Table 8.2. Continued

Criteria	Sub-criteria	Weights	Alternatives: Jeddah(1)	Alternatives: Dhahran(2)
X_6		Very important		
	y_{61}	Important	Fair	Fair
	y_{62}	Very important	Fair	Fair
	y_{63}	Rather unimportant	Poor	Good
	y_{64}	Moderately important	Good	Poor
X_7		Moderately important		
	y_{71}	Important	Good	Good
	y_{72}	Moderately important	Fair	Fair
	y_{73}	Rather unimportant	Good	Fair
	y_{74}	Very important	Fair	Good
	y_{75}	Important	Fair	Good
	y_{76}	Moderately important	Fair	Fair
	y_{77}	Very important	Fair	Fair
	y_{78}	Very important	Fair	Good
	y_{79}	Moderately important	Good	Poor
	y_{710}	Important	Fair	Fair

R_{ij}, where R_{ij} is given by:

$$R_{ij} = (r_{ij}, \mu_{R_{ij}}(r_{ij})) \quad , i = 1,2,\ldots,m$$
$$j = 1,2,\ldots,n$$

$$R_{ij} = \text{very good} = \{(0.8,0.0), (0.87,0.5), (0.95,1.0), (0.975,0.5), (1.0,0.0)\}$$

$$R_{ij} = \text{good} = \{(0.6,0.0), (0.7,0.5), (0.8,1.0), (0.9,0.5), (1.0,0.0)\}$$

$$R_{ij} = \text{fair} = \{(0.3,0.0), (0.4,0.5), (0.6,0.5), (0.5,1.0), (0.7,0.0)\}$$

$$R_{ij} = \text{poor} = \{(0,0.0), (0.1,0.5), (0.2,1.0), (0.3,0.5), (0.4,0.0)\}$$

$$R_{ij} = \text{very poor} = \{(0.0,0.0), (0.025,0.5), (0.05,1.0), (0.125,0.5), (0.25,0.0)\} .$$

The ratings for the two alternatives, Jeddah and Dhahran, are also shown in Table 8.2.

8.3.3 Ranking and preferability

The MAFDA computer program [105,106] is used to compute the final ranking and the degree of preference of each of the two alternatives, Jeddah and Dhahran. The results are shown in Tables 8.3 and 8.4, Figures 8.3 and 8.4. Table 8.3 lists the degree of membership, final rating, and the ranking of

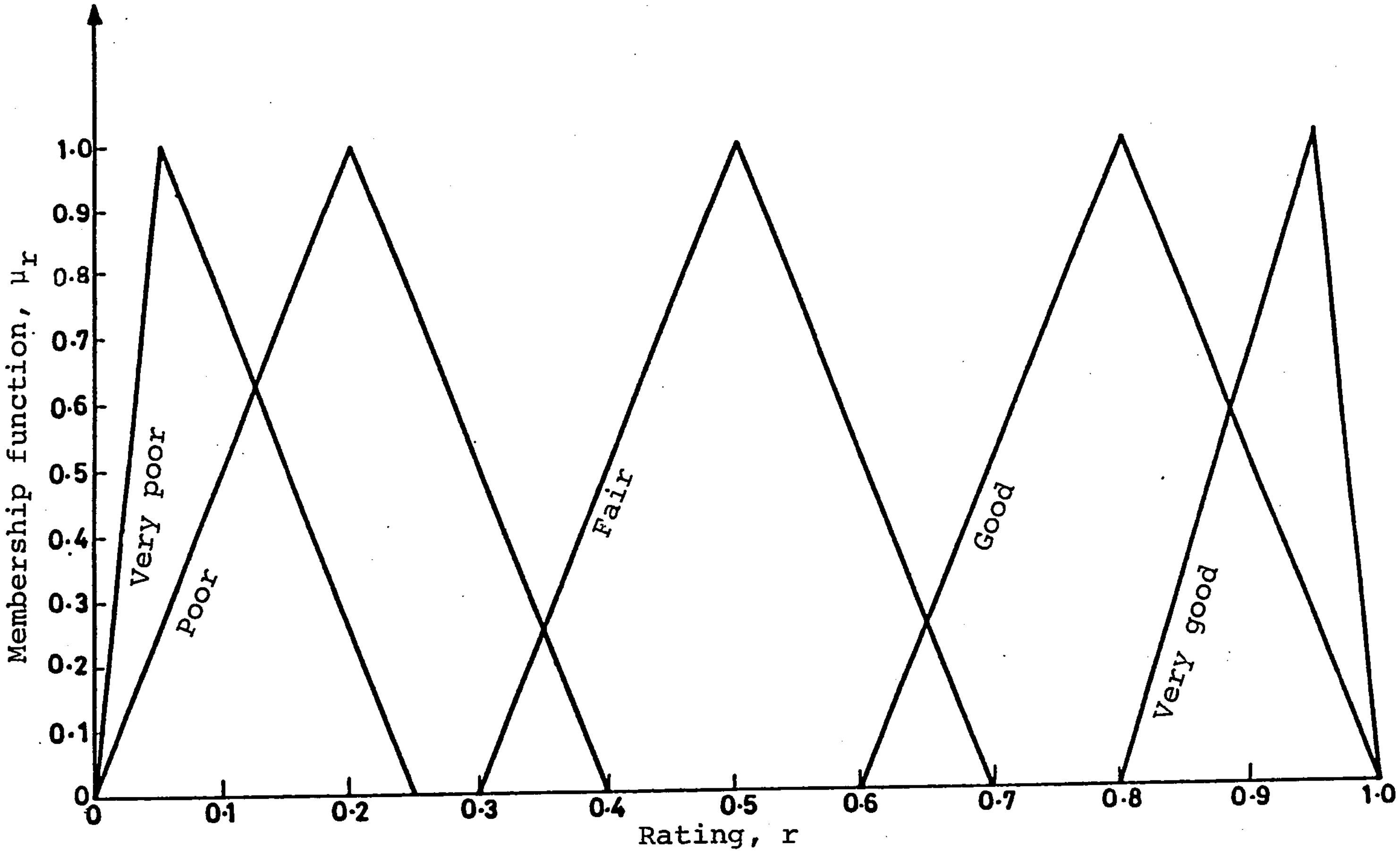


Figure 8.2. Membership function μ_r versus the rating r

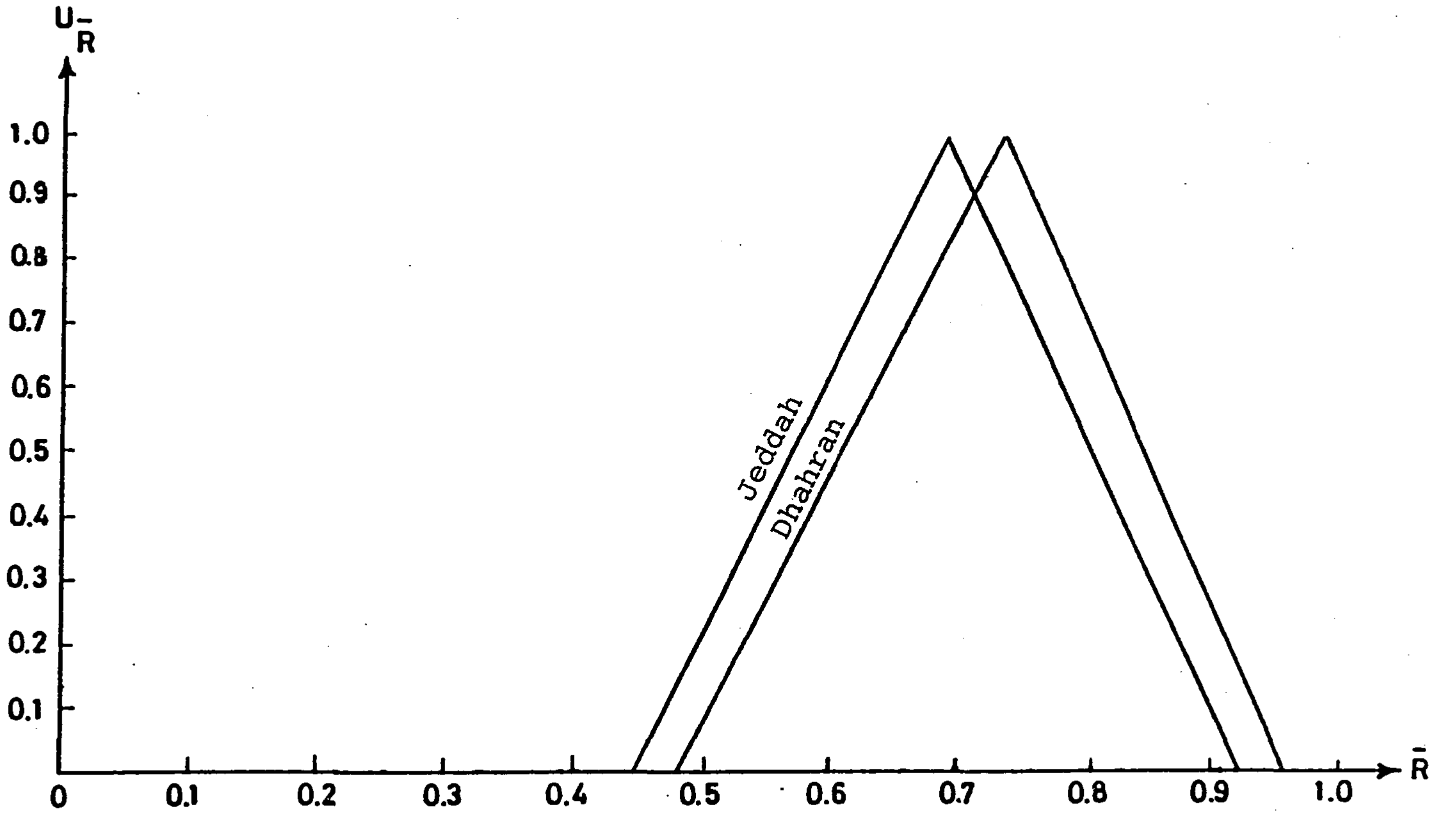


Figure 8.3. Final ranking membership functions for Jeddah and Dhahran sites in Saudi Arabia

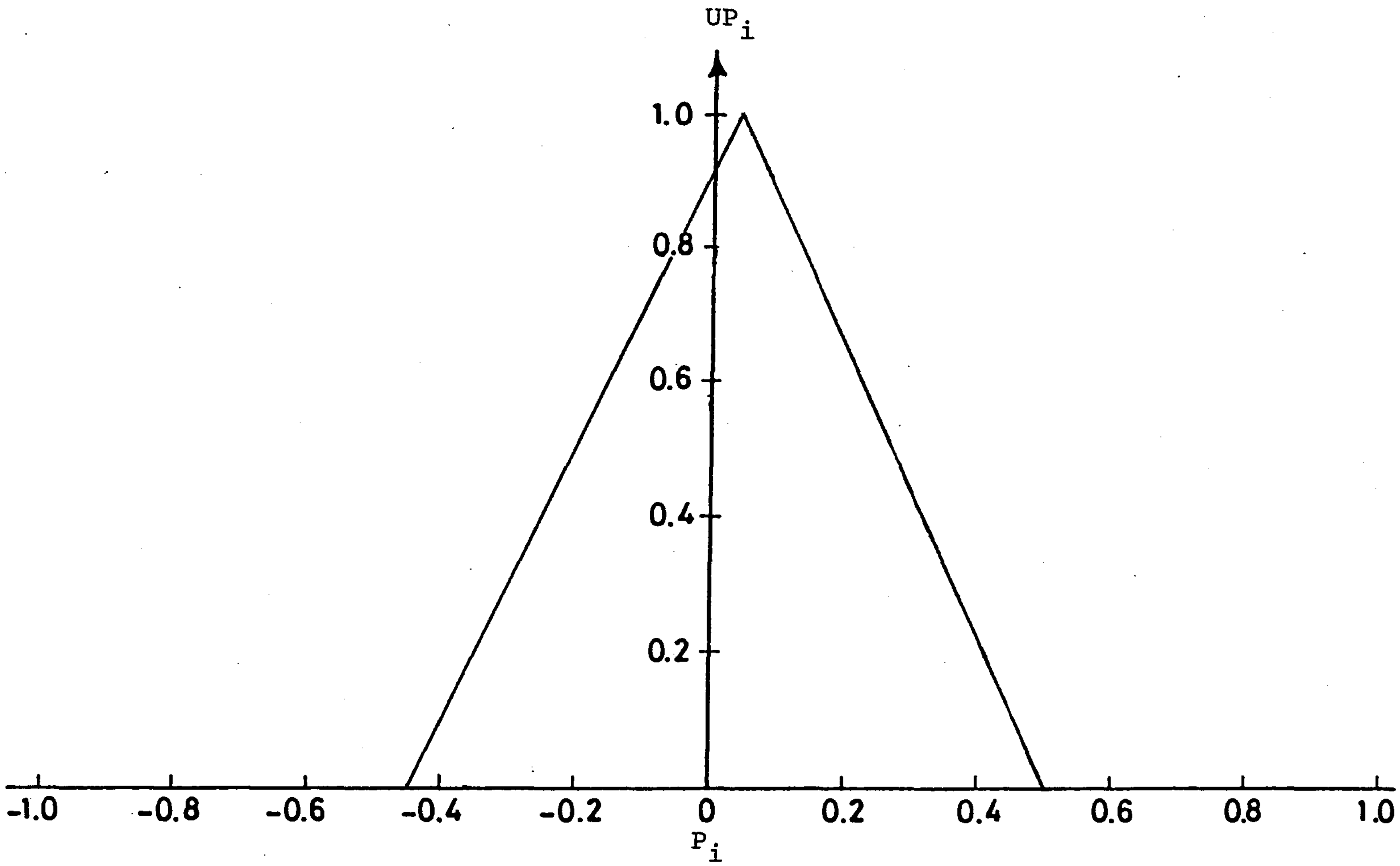


Figure 8.4. The membership function of preferability of Dhahran over Jeddah as site for nuclear power plant

Table 8.3. The membership, final rating, and the ranking of each criterion for Jeddah and Dhahran sites, Saudi Arabia

Criteria	The membership, $I^{(i)}$		Final rating		Ranking	
	Jeddah	Dhahran	Jeddah	Dhahran	Jeddah	Dhahran
Topography and oceanography	0.300	1.000	0.665	0.895	2	1
Geology, hydrology, and seismology	0.836	0.999	0.794	0.846	2	1
Meteorology	0.999	0.757	0.835	0.750	1	2
Transportation	1.000	0.807	0.874	0.809	1	2
Population	0.537	1.000	0.746	0.896	2	1
Cooling water	0.990	0.853	0.769	0.705	1	2
Construction, services, and domestic water	0.937	0.999	0.766	0.793	2	1

Table 8.4. The final membership, rating, and ranking of Jeddah and Dhahran sites

Alternative	Final membership	Final rating	Final ranking
Dhahran	1.00	0.823	1
Jeddah	0.929	0.795	2

each criterion for both Jeddah and Dhahran sites. Jeddah is preferred over Dhahran with respect to meteorology, transportation, and cooling water criteria, while Dhahran is preferred over Jeddah for the rest of the criteria. Table 8.4 lists the results of the final membership, rating, and ranking of the Jeddah and Dhahran sites, showing that Dhahran is a more suitable site than Jeddah. Finally, Figure 8.3 shows the final ranking of the Jeddah and Dhahran sites, while Figure 8.4 illustrates the membership function of preferability of Dhahran over Jeddah. From both figures, it is apparent that although the Dhahran site is preferred, the degree of preference over the Jeddah site is small.

9. SUMMARY AND CONCLUSIONS

A formal decision methodology is a sound approach for assisting in decision making on the selection of a nuclear research reactor facility and in reactor siting. A formal analysis, which has its limitations, is to be preferred over an informal, intuitive analysis which also has limitations. Such a formal analysis will focus on substantive issues and provide the basis for developing a compromise between conflicting objectives. All the critical issues in the selection of nuclear research reactor facilities and sites can be addressed within the decision analysis framework. Of special significance is the preferences of the decision maker and the treatment of the uncertainty associated with consequences of a decision.

In Chapter 2, a general review of appropriate research reactor types was conducted. In this review, the basic characteristics, concepts, safety considerations, design feature, material used, classification, and some common terms were discussed. Also, other types of facilities which are not appropriate because of complicated design features, used for specific purposes, excessive size, lower power density, maintenance problems, cost, and/or uncertainty of long-term utilization were discussed.

In Chapters 3 and 4, detail survey of available decision methods was carried out. These methods have been applied to various fields such as economics, business, management, war games, applied statistics, and operations research. Multi-attribute utility theory and fuzzy set theory were selected as a result of the survey. Historical background, the basic fundamentals of the utility theory, and the procedure for assessing utility functions were presented. The concept of multiattribute utility theory was also given.

In Chapter 5, structuring the decision problem was developed. It included the generation of the appropriate set of objectives, attributes, and subattributes which were used to evaluate and to indicate the degree of achievement for each alternative. Each category, attribute, and sub-attribute was defined to assure the uniqueness of each of them, to avoid confusion interpretation, and to eliminate double counting.

In Chapter 6, four different research reactor facilities were selected from which to choose the suitable one. They are the University of Michigan Ford Nuclear Reactor (FNR), the Massachusetts Institute of Technology Reactor (MITR), the Georgia Institute of Technology Research Reactor (GTRR), and the University of Wisconsin Nuclear Reactor (UWNR). They are pool, light water tank, heavy water tank and TRIGA reactor, respectively. A detailed study of each alternative was conducted to determine the level of impact of each of them corresponding to each category, attribute, and subattribute.

In Chapter 7, the multiattribute utility decision approach was applied to evaluate the four alternatives. Assumption verification, constructing preference curves, and assessment of the tradeoff constants were conducted to apply the "approach". UWNR facility, alternative four, has the highest utility value and hence it is the most preferred.

In Chapter 8, fuzzy set theory was employed to choose between two alternative sites in Saudi Arabia, namely Jeddah and Dhahran. The MAFDA computer program was used to compute the final ranking and the degree of preference of each of the two alternatives, Jeddah and Dhahran. The final ranking showed that the Dhahran site is preferred over the Jeddah site, but the degree of preference is small.

This study is of a preliminary nature. A more refined study of the alternative research reactors is recommended. Detailed data about cost, design features, safety considerations for each alternative have to be collected. Also, detailed data about local and import resources and site criteria have to be investigated.

The two decision methodologies adapted in this study can cover most decision problems related to the SNRC.

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12. APPENDIX A: RESEARCH REACTORS IN U.S.A. AND ABROAD BUILT BY U.S.A.

Pool, tank light water, tank heavy water, and TRIGA type are listed in this appendix. It includes research reactors in U.S.A. and research reactors abroad built by U.S.A., as well as operated and shut down reactors. Owner, designation, principal contractor, power, and date of start-up are given in the tables for each reactor.

Table 12.1. Pool type operated in U.S.A.

	Owner	Designation	Principal contractor	Power kw(+)	Date of startup
1.	Union Carbide Corporation Reactor	UCNR	AMF	5,000	1961
2.	Babcock & Wilcox Lynchburg Pool Reactor	LPR	B&W	1,000	1958
3.	Bulk Shielding Reactor	BSR	ORNL	2,000	1950
4.	Rhode Island Nuclear Science Center	--	GE	2,000	1964
5.	University of Kansas	Model 4180	BAC	10	1961
6.	University of Lowell	--	GE	1,000	1974
7.	University of Michigan	FNR	B&@	2,000	1957
8.	University of Missouri at Rolla	--	CW	200	1961
9.	N. Carolina State University	PUISTAR	AMF	1,000	1972
10.	Purdue University	--	Lockheed	1.0	1962

Table 12.1. Continued

	Owner	Desig-nation	Principal contractor	Power kw(+)	Date of startup
11.	State University of New York	PULSTAR	AMF	2,000	1961
12.	University of Virginia	UVAR	Owner + B&W	2,000	1960
13.	Worcester Poly-technic Institute	--	GE	10	1959

Table 12.2. Pool type operated abroad built by U.S.A.

	Country/Owner	Loca-tion	Princi-pal con-tractor	Power kw(+)	Start-up
1.	Australia, Seibers-dorf Research Center	Seibersdorf	AMF	5,000	1960
2.	Colombian Institute of Nuclear Affairs	Bogota	Lockheed	20	1965
3.	Germany, Society for Utilization of Nuclear Energy	Geesthacht	B&W	5,000	1958
4.	Greece, Atomic Energy Commission	Athens	AMF	1,000	1961
5.	Israel, Atomic Energy Commission	Nahal Soreq	AMF	5,000	1960
6.	Italy, Center for Military Application	Near Pisa	B&W	--	1963
7.	Italy, National Com-mittee for Nuclear Energy	Padua	AMF	Neglig	1971
8.	Italy, SORIN Nuclear Center	Saluggia	AMF	7,000	1959
9.	Pakistan, Atomic Energy Commission	Islamabad	AMF	5,000	1965
10.	Philippines, Atomic Energy Commission	Quezon City	GE	1,000	1963
11.	Portugal, National Lab of Engineering & Industrial Techology	Sacaven	AMF	1,000	1961

Table 12.2. Continued

	Country/Owner	Location	Principal contractor	Power kw(+)	Start-up
12.	Switzerland, Institute for Reactor Research	Wuerenlingen	ORNL	5,000	1957
13.	Turkey, Atomic Energy Commission	Istanbul	AMF	1,000	1962
14.	Venezuela Institute for Scientific Research	Caracas	GE	3,000	1960
15.	Brazil, University of São Paulo	São Paulo	B&W	5,000	1957
16.	Canada, McMaster University	Hamilton	AMF	5,000	1959
17.	Germany, Technical University of Munich	Munich	AMF	4,000	1957
18.	Iran, University of Tehran	Tehran	AMF	5,000	1967
19.	Netherlands, Delft Technical University	Delft	AMF	2,000	1962
20.	Uruguay, University of Montevideo	Montevideo	Lockheed	1,000	1973

Table 12.3. Pool type shutdown or dismantled in U.S.A.

	Owner	Designation	Principal contractor	Power kw(+)	Start-up	Shut-down
1.	Babcock & Wilcox Nuclear Development Center Test Reactor	BAWTR	B&W	6,000	1964	1971
2.	Industrial Reactor Laboratories Inc.	--	AFM	5,000	1958	1975
3.	Battelle Memorial Institute	BRR	AMF	2,000	1956	1974
4.	Curtiss-Wright Nuclear Research Laboratory of Commonwealth of Pennsylvania	--	Owner	1,000	1958	1966

Table 12.3. Continued

	Owner	Designation	Principal contractor	Power kw(+)	Start-up	Shut-down
5.	DOE Demonstration Reactor	Demo Reac	Lockheed	10	1969	1969
6.	European-Asian Exhibit Program	--	Lockheed	10	1963	1969
7.	Lockheed Aircraft Corp.	--	Lockheed	Neglig	1960	1960
8.	Louisiana State University Nuclear Science Center	SNARE	Sandia	2	1965	1966
9.	Radiation Effect Reactor	RER	Lockheed	3,000	1958	1970
10.	Shield Test and Irradian Reactor	STIR	AI	1,000	1961	1972
11.	Leland Stanford University	--	GE	10	1959	1974

Table 12.4. Tank type operated in U.S.A.

		Tank type	Designation	Principal contractor	Power kw(+)	Start-up
1.	Brookhaven Medical Research Reactor (DOE)	H_2O	BMRR	Daystrom	5,000	1959
2.	Nat. Bureau of Standards Reactor	D_2O	NBSR	NBS-BVR	10,000	1967
3.	Oak Ridge Research Reactor (DOE)	H_2O	ORR	ORNL	30,000	1958
4.	Omega West Reactor (DOE)	H_2O	OWR	LANL	8,000	1956
5.	Biological Research Reactor (DOE)	H_2O	JANUS	ANL	200	1964
6.	Cornell University Zero Power Reactor	H_2O	ZPR	Vitro	Neglig	1962
7.	Manhattan College	H_2O	--	AMF	Neglig	1964

Table 12.4. Continued

	Tank type	Designation	Principal contractor	Power kw(+)	Start-up
8. Massachusetts Institute of Technology	H_2O	MITR	ACF or AC	5,000	1958
9. University of Missouri	H_2O	MURR	Owner + IC	10,000	1966
10. Georgia Tech. Research Reactor	D_2O	GTRR	GNEC	10,000	1964

Table 12.5. Tank type operated abroad built by U.S.A.

	Tank type	Location	Principal contractor	Power kw(+)	Start-up
1. Japan Atomic Energy Research Institute	D_2O	Tokai	AMF	10,000	1960
2. Netherlands, Reactor Center	H_2O	Petten	AC	45,000	1961
3. South Africa Atomic Energy Board	H_2O	Pelindaba	AC	20,000	1965
4. Sweden, Studsvik Energiteknik	H_2O	Studsvik	AB	50,000	1960
5. Italy, National Committee for Nuclear Energy	D_2O	Ispra	AC	5,000	1959

Table 12.6. Tank type shutdown or dismantled in U.S.A.

Owner	Tank type	Designation	Principal contractor	Power kw(+)	Start-up	Shut-down
1. Ames Laboratory Research Reactor (DOE)	D_2O	ALRR	AMF	5,000	1955	1977
2. Argonne Research Reactor (DOE)	D_2O	CP-5	ANL	5,000	1954	1979
3. Sandia Engineering Reactor (DOE)	H_2O	SER	Sandia	5,000	1961	1970
4. Argonne CP-3	D_2O	CP-3	Met. Lab.	300	1944	1963
5. Brookhaven Neutron Source Reactor No. 1 (DOE)	H_2O	SCHIZO	BNL	100	1958	1970
6. Brookhaven Neutron Source Reactor No. 2 (DOE)	H_2O	PHRENIC	BNL	100	1965	1970
7. Livermore Reactor	H_2O	LPTR	FW	3,000	1957	1980
8. Low Intensity Test Reactor	H_2O	LITR	ORNL	3,000	1950	1965
9. Tower Shielding Reactor No. 1	H_2O	TSR-1	ORNL	500	1954	1958

Table 12.7. TRGIA type operated in U.S.A.

	Owner	Designation	Con-tractor	Power	Start-up
1.	Aerotest Operation, Inc.	AGNIR	GA	250	1965
2.	Dow Chemical Co.	TRIGA-MKI	GA	100	1967
3.	General Atomic Company TRIGA-MKF	TRIGA-MKF	GA	1,500	1960
4.	General Atomic Company TRIGA-MKI	TRIGA-MKI	GA	250	1958
5.	Neutron Radiography Facility (DOE)	NRAD	ANL	2,500	1977
6.	Northrop Corporate Laobratories	TRIGA-MKF	GA	1,000	1963
7.	Omaha Veterans Administration Hospital	TRIGA-MKI	GA	18	1959
8.	U.S. Geological Survey Laboratory	TRIGA-MKI	GA	1,000	1969
9.	Univ. of Arizona	TRIGA-MKI	GA	100	1958
10.	Univ. of California, Berkeley	TRIGA-MKIII	GA	1,000	1967
11.	Univ. of California, Irvine	TRIGA-MKI	GA	250	1969
12.	Columbia Univ.	TRIGA-MKII	GA	250	Licensed
13.	Cornell Univ.	TRIGA-MKII	GA	100	1962
14.	Univ. of Illinois	TRIGA-MKII	GA	1,500	1960
15.	Kansas State Univ.	TRIGA-MKII	GA	250	1962
16.	Univ. of Maryland	TRIGA-Tank	GA	250	1974
17.	Michigan State U.	TRIGA-MKI	GA	250	1969
18.	Oregon State Univ.	TRIGA-MKII	GA	1,000	1967
19.	Penn State TRIG Reactor	TRIGA-Pool	GA	1,000	1965
20.	Reed College	TRIGA-MKI	GA	250	1968
21.	Texas A&M U. (Nuclear Sci. Center Reactor)	NSCR	GA	1,000	1961

Table 12.7. Continued

	Owner	Designation	Con-tractor	Power	Start-up
22.	Uni. of Texas at Houston	TRIGA-MKI	GA	250	1963
23.	University of Utah	TRIGA-MKI		250	1975
24.	Washington State University	WSTR	GA	1,000	1967
25.	U. of Wisconsin	TRIGA	GA	1,000	1967

Table 12.8. TRIGA type operated abroad built by U.S.A.

	Owner	Loca-tion	Con-tractor	Power	Start-up
1.	Japan Atomic Energy Research Institute	Tokai-Mura	GA	300	1975
2.	Romania, Institute for Nuclear Tech-nologies	Bucharest	GA	500	1979
3.	Romania, Institute for Nuclear Tech-nologies	Bucharest	GA	14,000	1979
4.	England, Imperial Chemical Industries	Billingham	GE	250	1971
5.	Indonesia, National Atomic Energy Agency	Bandung	GA	1,000	1964
6.	Indonesia, National Atomic Energy Agency	Jogjakarta	GA	250	1979
7.	Italy, National Committee for Nuclear Energy	Rome	GA	1,000	1960
8.	Korea, Atomic En-ergy Research In-stitute	Seoul	GA	250	1962
9.	Korea, Atomic En-ergy Research In-stitute	Seoul	GA	2,000	1972

Table 12.8. Continued

Owner	Location	Contractor	Power	Start-up
10. Mexico, National Commission for Nuclear Energy	Salazar	GA	1,000	1968
11. Thailand, Office of Atomic Energy for Peace	Bangkok	GA	2,000	1977
12. Turkey, Technical Univ. of Istanbul	Istanbul	GA	250	1979
13. Yugoslavia, Josef Stefan Nuclear Inst.	Ljubljana	GA	250	1966
14. Zaire (Reg. Center for Nuclear Studies)	Kinsha	GA	1,000	1959
15. Austria, Vienna Polytechnic Inst.	Vienna	GA	250	1962
16. Brazil, Univ. of Minas Gerais	Belo Horizonte	GA	250	1960
17. China, Republic of (Nat. Tsing-Hua U.)	Hsinchu	GA	1,000	1977
18. Finland, Institute of Technology	Helsinki	GA	250	1962
19. Germany, Assoc. of Radiation Research	Munich	GA	1,000	1972
20. Germany, Institute for Nuclear Medicine	Heidelberg	GA	250	1966
21. Germany, Johannes Gutenberg Univ. of Mainz	Mainz	GA	100	1965
22. Germany, Medical College of Hanover	Hanover	GA	250	1972
23. Italy, Univ. of Pavia	Pavia	GA	250	1965
24. Japan, Musahi College of Technology	Tokyo	GA	100	1963
25. Japan, Rikkyo Univ.	Yokosuka	GA	100	1961
26. Zaire, University of Lovanium	Kinshasa	GA	1,000	1959

Table 12.9. TRIGA type shutdown or dismantled in U.S.A.

Name	Designation	Contractor	Power kw(+)	Start-up	Shutdown
1. General Atomic Co. (World Agricultural Fair)	TRIGA-MKI	GA	50	1960	1960
2. Torrey Pines, TRIGA-MKIII Reactor	TRIGA-MKIII	GA	1,500	1966	1973
3. Puerto Rico Nuclear Center (DOE)	TRIGA-FLIP	GA	2,000	1972	1976

Table 12.10. Pool, tank, and TRIGA type shutdown or dismantled abroad built by U.S.A.

	Reactor type	Location	Principal contractor	Power kw(+)	Start-up	Shutdown
1. Spain, Nuclear Energy Board	Pool	Moncloa	GE	3,000	1958	1976
2. Denmark, Risö National Laboratory (DR-2)	Tank-H_2O	Risö	FW	5,000	1958	1975
3. Vietnam, Institute of Nuclear Research	TRIGA	Dalat	GA	250	1963	1973

13. APPENDIX B: MULTIATTRIBUTE UTILITY DECISION ANALYSIS PROGRAM

13.1. Introduction

A computer program is developed to simplify the many computational difficulties involved in the multiattribute utility theory. The program calculates the following:

(1) the coefficients for each category, attribute, and subattribute which represent the best fit for an exponential curve;

(2) the values of ks, scaling constants, which are needed to construct the multiattribute utility function; and

(3) the utility function for each alternative.

The program is developed for the selection between nuclear research reactor facilities. Yet, it has a great degree of flexibility so that it can be used for solution of a variety of decision problems of the same nature as the problem analyzed here.

13.2. Data Input Description

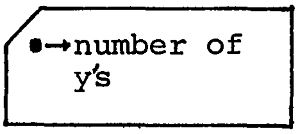
This section describes the data which must be input to the program.

<u>Input Group 1</u>

Number of Cards: 1

Format : integer

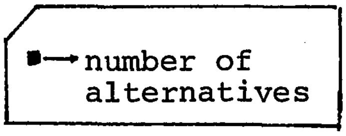
Input data : number of attributes (ys)



Input Group 2

Number of cards: 1

Format : integer



Input data : number of alternatives

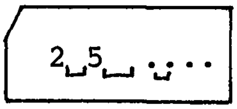
Input Group 3

Number of cards: 1 or 2

Format : integer

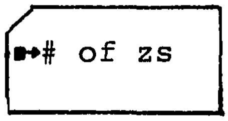
Input data : number of subattributes (xs) in each attribute (ys)

Example: If the first attribute contains 2 subattributes and the second attribute contains 5 subattributes, so we write and so on.



Input Group 4

Number of cards: 1



Format : integer

Input data : number of categories (zs)

Input Group 5

Number of cards: 1 or 2

Format : integer

Input data : number of attributes (ys) in each category (z)

Example: If the first category

contains 3 attributes and the second category contains 6 attributes, so we write and so on.

```
3␣6␣.. .
```

Input Group 6

Number of cards: equal to the number of subattributes (xs)

Format : real

Input data : best, worst, certainty equivalent, value alternative one, two, ... for each subattribute and each subattribute should be in a separate card.

```
best␣worst␣C.E.␣AH1␣AH2    ....
```

Input Group 7

Number of cards: (number of subattributes in each attribute) x (number of attributes)

Format : real

Input data : The number of card sets is equal to the number of attributes. There is a set for each attribute which represents the tradeoff between its subattributes and the number of cards in this set is equal to the number of subattributes. Each set of cards contains the following:

1. The first card has the highest value of the tradeoff constant (given by decision maker) of the subattribute and its order between the rest subattributes in the attribute
2. The rest cards, each contains the level of x (of highest subattribute) that represents the tradeoff constant of the subattribute and the cards should be in order starting with the first subattribute in that attribute.

Example: Attribute y has 3 subattributes (x_1, x_2, x_3). x_2 is the most important subattribute. Its tradeoff constant is $K_2 = 0.95$ given by decision maker.

$$K_1 = K_2\mu_2(x_2'),\ K_3 = K_2\mu_2(x_2'')$$

where x_2', x_2'' are some level of x_2 and equal to $x_2' = 30$, $x_2'' = 25$.

1. First card will have 0.95 and 2, which is the order of this subattribute

2. The rest cards, 2 in this case, will have 30 in the first card of the rest and 25 in the second card of the rest. Note they are in order.

First card

0.95␣2

Other cards

30

25

Input Group 8

Number of cards: equal to the number of attributes (ys)

Format : real

Input data : best, worst, certainty equivalent for each attribute. Each attribute should be in a separate card.

best␣worst␣C.E.

Input Group 9

Number of cards: (number of attributes in each category) x (number of categories)

Format : real

Input data : the same procedure as group 7. They are just for attributes instead of subattributes.

Input Group 10

Number of cards: equal to the number of categories

Format : real

Input data : best, worst, certainty equivalent for each category. Each category should be in a separate card.

```
best␣worst␣C.E.
```

Input Group 11

Number of cards: equal to the number of categories

Format : real

Input data : the same procedure as group 7. They are just for categories instead of subattributes.

13.3. Program Listing and Output

```
//A238  JOB  $$$$$,WALEED
***ROUTE PRINT LOCAL
***JOBPARM  FORMS=1005
//S1 EXEC P8000CG,TIME=(,5)
//PASC.SYSIN  DD  *
      PROGRAM MAIN (INPUT,OUTPUT) ;
      CONST
      TOL = 0.001 ;
      DECR  = 0.012 ;
      INCR = 0.012 ;
      ITRNUM = 100 ;
      VAR
      NX : ARRAY (.1..100.) OF INTEGER ;
      STORE :ARRAY (.1..100.) OF INTEGER ;
      NZ : ARRAY (.1..100.) OF INTEGER ;
      X : ARRAY (.1..100,1..20.) OF REAL;
      Y : ARRAY (.1..20,1..20.) OF REAL;
      Z : ARRAY (.1..20,1..20.)  OF REAL ;
      YF : ARRAY (.1..20.)  OF REAL;
      ZF : ARRAY (.1..20.) OF REAL ;
      K    :INTEGER ;
      TEMPSTR   : INTEGER ;
      NUMZ : INTEGER ;
      U,J,I,UPPOS  : INTEGER ;
      MAX,NUMBER :INTEGER;
      UPPERLIM,LOWERLIM,K1 :REAL   ;
      ZSKS    : BOOLEAN ;
      YSKS   : BOOLEAN ;
      TEMP : REAL ;



  A   PROCEDURE KVALUES  ;
      VAR
      Q,I :INTEGER ;
      RIGHT,LEFT,BETWEEN,MID  :REAL;
  A   BEGIN
      IF (YSKS) OR ( ZSKS )  THEN
      NX(.U.) := TEMPSTR ;
      BETWEEN := 1;   UPPERLIM := 2; LOWERLIM:= -1 ;
      IF TEMP >1 THEN
      BEGIN
      UPPERLIM :=-0.02 ; LOWERLIM :=-1;
      END
      ELSE
      BEGIN
      UPPERLIM :=1 ;
      LOWERLIM:=0.02;
      END;
```

```
Q:= 1;
WHILE (Q <= 100) AND (ABS(BETWEEN)>0.0001) DO
BEGIN
MID := (UPPERLIM+LOWERLIM)/2  ;
RIGHT := 1;
BETWEEN := 1;
LEFT:=1;
FOR I:= 1 TO NX(.U.) DO
BEGIN
IF (NOT YSKS) AND (NOT ZSKS) THEN
RIGHT := RIGHT*(1+UPPERLIM*X(.I+UPPOS,20.))
ELSE
IF NOT ZSKS THEN
RIGHT := RIGHT *(1+UPPERLIM*Y(.I,20.))
ELSE   RIGHT := RIGHT *(1+UPPERLIM*Z(.I,20.)) ;
END ;
RIGHT := RIGHT  - 1 - UPPERLIM  ;
FOR I:= 1 TO NX(.U.) DO
BEGIN
IF (NOT YSKS) AND (NOT ZSKS) THEN
BETWEEN :=BETWEEN *(1+MID*X(.I+UPPOS,20.))
ELSE
IF NOT ZSKS THEN
BETWEEN := BETWEEN*(1+MID*Y(.I,20.))
ELSE   BETWEEN := BETWEEN*(1+MID*Z(.I,20.)) ;
END ;
BETWEEN := BETWEEN - 1 - MID ;
FOR I:= 1 TO NX(.U.) DO
BEGIN
IF (NOT YSKS) AND (NOT ZSKS) THEN
LEFT := LEFT *(1+LOWERLIM*X(.I+UPPOS,20.))
ELSE
IF NOT ZSKS THEN
LEFT := LEFT*(1+LOWERLIM*Y(.I,20.)   )
ELSE     LEFT := LEFT*(1+LOWERLIM*Z(.I,20.))  ;
END ;
LEFT := LEFT - 1 - LOWERLIM ;
IF ((LEFT*BETWEEN)>0) AND ((BETWEEN*RIGHT)<0) THEN
LOWERLIM := MID
ELSE
BEGIN
IF ((LEFT*BETWEEN)<0) AND ((BETWEEN*RIGHT)>0) THEN
UPPERLIM := MID
ELSE
BEGIN
IF((LEFT*BETWEEN<0)AND(BETWEEN*RIGHT<0))
OR ((LEFT*BETWEEN>0)AND(BETWEEN*RIGHT>0)) THEN
BEGIN
LOWERLIM := LOWERLIM + INCR ;
UPPERLIM := UPPERLIM- DECR ;
```

```
END;
END
END;
Q:=Q+1  ;
END;
IF ABS(BETWEEN) <0.0001 THEN
K1 := MID
ELSE
WRITELN (' K1 NOT FOUND');
END;

PROCEDURE NOTUNITY (U,UPPOS   :INTEGER    ) ;
VAR
STOP   : BOOLEAN ;
Q,I,J,N,POS,R   : INTEGER ;
RESULT   : REAL ;
  BEGIN
KVALUES ;
Y(.U,19.) := K1 ;
FOR POS:=1 TO MAX DO
BEGIN
RESULT:=1;
FOR I:=1 TO NX(.U.) DO
RESULT:=RESULT*(1+K1*X(.I+UPPOS,20.)*X(.I+UPPOS,POS+9.));
Y(.U,POS+9.):=(RESULT-1)/K1;
END;
END;


PROCEDURE FINDVALUES    ;
  VAR
N,M,I,J,K,R    : INTEGER ;
FACTOR,U1,U2,U3,X1,X2,X3,C1,B1   :   REAL ;
UPPER,LOWER,MID,LEFT,BETWEEN,RIGHT  : REAL ;
BEGIN
YSKS := FALSE ;
ZSKS := FALSE ;
READ(MAX); (*MAX NUMBER OF ALTERNATIVES*)
READLN  ( NUMBER )  ;   (*NUMBER OF Y'S *)
FOR R := 1 TO NUMBER DO
BEGIN
READ (NX(.R.));
STORE (.R.) := NX(.R.) ;
END;
READLN;
READ (NUMZ) ;
FOR R:= 1 TO NUMZ DO
READ (NZ(.R.)) ;
READLN ;
U:=1; N:=0;
FOR M:=1 TO NUMBER DO
```

```
BEGIN
FOR J:=1 TO NX(.U.) DO
BEGIN
FOR I:=1 TO 3 DO
BEGIN
READ(X(.J+N,I.));

END ;
X(.J+N,4.)  := 1   ;
X(.J+N,5.):=0;
X(.J+N,6.):=0.5;
U1 := X(.J+N,4.) ;
U2 := X(.J+N,5.)  ;
U3  := X(.J+N,6.) ;
X1 := X(.J+N,1.) ;
X2  := X(.J+N,2.)  ;
X3  := X(.J+N,3.)  ;
BETWEEN:=10;I:=1;UPPER:=1.2;LOWER:=-1;
WHILE (I<= 100) AND  (ABS(BETWEEN)>0.0001)  DO
BEGIN
MID := (UPPER + LOWER )/2 ;
RIGHT :=(((U1-U2)/((EXP(UPPER*X1))-(EXP(UPPER*X2))))*
((EXP(UPPER*X2))-(EXP(UPPER*X3))))-(U2-U3);
BETWEEN := (((U1-U2)/((EXP(MID*X1))-(EXP(MID*X2))))*
((EXP(MID*X2))-(EXP(MID*X3)))) -(U2-U3) ;
LEFT := (((U1-U2)/(EXP(LOWER*X1)-EXP(LOWER*X2)))*
((EXP(LOWER*X2))-(EXP(LOWER*X3 ))))-(U2-U3) ;
IF ((LEFT*BETWEEN)>0) AND (BETWEEN * RIGHT< 0) THEN
LOWER := MID
ELSE
BEGIN
IF (LEFT*BETWEEN< 0) AND (BETWEEN*RIGHT>0) THEN
UPPER := MID
ELSE
BEGIN
IF((LEFT*BETWEEN<0)AND(BETWEEN*RIGHT<0))
OR((LEFT*BETWEEN>0)AND(BETWEEN*RIGHT>0)) THEN
BEGIN
UPPER := UPPER - DECR ;
LOWER := LOWER + INCR ;
END ;
END
END ;
I := I+ 1 ;
END ;
IF  ABS(BETWEEN) <0.0001  THEN
C1 := MID ;
X(.J+N,9.) := C1 ;
X(.J+N,8.) := (U1-U2)/((EXP(C1*X1))-(EXP(C1*X2))) ;
```

```
B1 := X(.J+N,8.)   ;
X(.J+N,7.) := U1 -(B1*(EXP(C1*X1)))       ;

FOR K:=1 TO MAX DO
BEGIN
READ(FACTOR);

X(.J+N,K+9.):=X(.J+N,7.)+((X(.J+N,8.))*(EXP(C1*FACTOR)));
END;
READLN;
WRITELN ;
END;
N:=N+NX(.U.);
U:=U+1;
END;
END  ;


PROCEDURE UNITY ;
VAR
I,J,M,N,V,R,STRONGX,UPPER : INTEGER ;
K,GIVENX,A,B,C,FLAG : REAL ;
 BEGIN
UPPOS := 0 ;  U := 1 ;
WHILE U <= NUMBER DO
BEGIN
IF  NX(.U.) <> 1 THEN
BEGIN
READLN(K,STRONGX);

A :=X(.STRONGX+UPPOS,7.) ;
B:=X(.STRONGX+UPPOS,8.);
C:=X(.STRONGX+UPPOS,9.);
FOR I:=1 TO NX(.U.) DO
BEGIN
IF(I<>STRONGX) THEN
BEGIN
            READ (GIVENX )  ;

X(.I+UPPOS,20.):= K*(A+B*(EXP(C*GIVENX)))          ;

END
ELSE
X(.UPPOS+I,20.):=K;
END;
```

```
IF  NX(.U.)  <> 1 THEN
BEGIN
TEMP:=0;
FOR V:=1 TO NX(.U.) DO
TEMP:=X(.V+UPPOS,20.)+TEMP;
IF TEMP  = 1 THEN
BEGIN
FOR M:=1 TO MAX DO
BEGIN
FLAG:=0;
FOR N:=1 TO NX(.U.) DO
FLAG:=FLAG+(X(.N+UPPOS,M+9.)*X(.N+UPPOS,20.));
Y(.U,M+9.):=FLAG;
END;
Y(.U,19.):=0;
END
ELSE NOTUNITY (U,UPPOS ) ;
END
ELSE
BEGIN
FOR  R:=1 TO MAX DO
Y(.U,R+9.):=X(.UPPOS+1,R+9.);
END ;
READLN ;
  END
ELSE
BEGIN
X(.UPPOS+1,20.) := 0;
UPPER := ( MAX+9)  ;
FOR M:= 1 TO UPPER DO
Y(.U,M.) := X(.UPPOS+1,M.)  ;
Y(.U,19.) := 0;
END ;
UPPOS:=UPPOS+NX(.U.);
U:=U+1;
END;
A  END ;
A  PROCEDURE YSUTILITITY ;
VAR
DOWN  : INTEGER ;
STOP : BOOLEAN ;
POS,J,I,STRONGESTY,V,M,N,Q :INTEGER;
U : INTEGER ;
```

```
K,A,B,C,GIVENY,FLAG,RESULT,U1,U2,U3,Y1,Y2,Y3,C1,B1 :REAL;
UPPER,LOWER,MID,LEFT,BETWEEN,RIGHT   : REAL ;
BEGIN

FOR J:= 1 TO  NUMBER DO
BEGIN
IF NX(.J.) <> 1 THEN
BEGIN
FOR I := 1 TO 3 DO
READ (Y(.J,I.));
READLN;
Y(.J,4.) := 1 ;
Y(.J,5.) := 0 ;
Y(.J,6.) := 0.5  ;
U1 :=Y(.J,4.)  ;
U2 := Y(.J,5.)  ;
U3  := Y(.J,6.)  ;
Y1 :=  Y(.J,1.)  ;
Y2 :=  Y(.J,2.)  ;
Y3 := Y(.J,3.)  ;
BETWEEN:=10;I:=1;UPPER:=1.2;LOWER:=-1;
WHILE (I<= 100) AND  (ABS(BETWEEN)>0.0001)  DO
BEGIN
MID := (UPPER + LOWER )/2 ;
RIGHT :=(((U1-U2)/((EXP(UPPER*Y1))-(EXP(UPPER*Y2))))*
((EXP(UPPER*Y2))- (EXP(UPPER*Y3))))-(U2-U3) ;
BETWEEN :=(((U1-U2)/((EXP(MID*Y1))-(EXP(MID*Y2))))*
-(EXP(MID*Y3))))-(U2-U3) ((EXP(MID*Y2))     ;
LEFT := (((U1-U2)/(EXP(LOWER*Y1)-EXP(LOWER*Y2)))*
((EXP(LOWER*Y2))-(EXP(LOWER*Y3))))-(U2-U3) ;
IF ((LEFT*BETWEEN)>0) AND (BETWEEN * RIGHT< 0) THEN
LOWER := MID
ELSE
BEGIN
IF (LEFT*BETWEEN< 0) AND (BETWEEN*RIGHT>0) THEN
UPPER := MID
ELSE
BEGIN
IF((LEFT*BETWEEN<0)AND(BETWEEN*RIGHT<0)) OR
((LEFT*BETWEEN>0)AND(BETWEEN*RIGHT>0)) THEN
BEGIN
LOWER := LOWER + INCR ;
UPPER := UPPER - DECR ;
END ;
END
END ;
I := I+ 1 ;
END ;
IF  ABS(BETWEEN) <0.0001  THEN
```

```
C1  := MID ;
Y(.J,9.) := C1 ;
Y(.J,8.):=(U1-U2)/((EXP(C1*Y1))-(EXP(C1*Y2)))        ;
B1  := Y(.J,8.)   ;
Y(.J,7.):=U1-(B1*(EXP(C1*Y1)))          ;
END
END;
DOWN  := 0 ;
FOR U:= 1 TO NUMZ DO
BEGIN
READLN (K,STRONGESTY);
A:= Y(.STRONGESTY+DOWN,7.);
B:= Y(.STRONGESTY+DOWN,8.) ;
C:= Y(.STRONGESTY+DOWN,9.);
FOR  J:= 1 TO NZ(.U.) DO
BEGIN
IF (J<>STRONGESTY) THEN
BEGIN
READLN (GIVENY);
Y(.J+DOWN,20.) := K*(A+B*EXP(C*GIVENY));
END
ELSE
Y(.J+DOWN,20.) := K;
END;
TEMP := 0;
FOR V := 1 TO NZ(.U.) DO
TEMP:=TEMP+ Y(.V+DOWN,20.) ;
IF TEMP = 1 THEN
BEGIN
FOR M := 1 TO MAX DO
BEGIN
FLAG := 0;
FOR N:= 1 TO NZ(.U.) DO
FLAG := FLAG+(Y(.N+DOWN,M+9.)*Y(.N+DOWN,20.)) ;
Z(.N,M+9.) := FLAG ;
END;
Z(.U,19.) := 0;
END
ELSE
BEGIN
YSKS  := TRUE ;
TEMPSTR := NZ(.U.) ;
KVALUES  ;
Z(.U,19.) := K1 ;
END ;
FOR POS := 1 TO MAX DO
BEGIN
RESULT := 1;
FOR  J:= 1 TO NZ(.U.) DO
RESULT := RESULT*(1+K1*Y(.J+DOWN,20.)*Y(.J+DOWN,POS+9.));
```

```
Z(.U,POS+9.) := (RESULT-1)/K1  ;
END ;
DOWN := DOWN + NZ(.U.) ;
END;
END;

PROCEDURE ZSUTILITY ;
VAR
J,N,M,I,V,POS,STRONGZ   : INTEGER ;
U1,U2,U3,Z1,Z2,Z3,BETWEEN,UPPER,LOWER,MID,LEFT,RIGHT : REAL ;
K,GIVENZ,FLAG,RESULT,A,B,C,A1,B1,C1  :REAL;
BEGIN
YSKS := FALSE ; ZSKS := TRUE ;
FOR J:=1 TO NUMZ DO
BEGIN
 IF NZ(.J.) <>1 THEN
BEGIN
FOR I:=1 TO 3 DO
READ(Z(.J,I.));
READLN;
Z(.J,4.):= 1;
Z(.J,5.):= 0;
Z(.J,6.):= 0.5;
U1:=Z(.J,4.);
U2:=Z(.J,5.);
U3:=Z(.J,6.);
Z1:=Z(.J,1.);
Z2:=Z(.J,2.);
Z3:=Z(.J,3.);
   BETWEEN:=10; I:=1; UPPER:=1.2; LOWER:=-1;
WHILE (I<= 100) AND  (ABS(BETWEEN)>0.0001)  DO
BEGIN
MID:=(UPPER+LOWER)/2;
     RIGHT:=(((U1-U2)/((EXP(UPPER*Z1))-(EXP(UPPER*Z2))))*((EXP(UPPER*Z2))
```

```
                    -(EXP(UPPER*Z3)))))-(U2-U3);
            LEFT :=(((U1-U2)/(EXP(LOWER*Z1)-EXP(LOWER*Z2)))*((EXP(LOWER*Z2))-
                    (EXP(LOWER*Z3)))))-(U2-U3);
            BETWEEN:=(((U1-U2)/((EXP(MID*Z1))-(EXP(MID*Z2))))*((EXP(MID*Z2))-
                      (EXP(MID*Z3)))))-(U2-U3);
       IF((LEFT*BETWEEN)>0)AND((BETWEEN*RIGHT)<0) THEN
       LOWER:=MID
       ELSE
       BEGIN
       IF(LEFT*BETWEEN<0)AND(BETWEEN*RIGHT>0) THEN
       UPPER:=MID
       ELSE
       BEGIN
       IF((LEFT*BETWEEN<0)AND(BETWEEN*RIGHT<0))OR((LEFT*BETWEEN>0)AND
       (BETWEEN*RIGHT>0))THEN
       BEGIN
       UPPER := UPPER - DECR ;
       LOWER := LOWER + INCR ;
       END;
       END;
       END;
       I:=I+1;
       END;
       IF ABS(BETWEEN)<0.0001 THEN
       C:=MID;
       Z(.J,9.):=C;
       Z(.J,8.):=(U1-U2)/((EXP(C*Z1))-(EXP(C*Z2)));
       B:=Z(.J,8.);
       Z(.J,7.):=U1-(B*(EXP(C*Z1)));
       END;
       END;
       READLN (K,STRONGZ);
       A1:=Z(.STRONGZ,7.);
       B1:=Z(.STRONGZ,8.);
       C1:=Z(.STRONGZ,9.);
```

```
FOR J:=1 TO NUMZ DO
BEGIN
IF (J<>STRONGZ) THEN
BEGIN
READLN(GIVENZ);
Z(.J,20.):=K*(A1+B1*EXP(C1*GIVENZ))
END
ELSE
Z(.J,20.):=K;
END;
TEMP:=0;
FOR V:=1 TO NUMZ DO
TEMP:=TEMP+Z(.V,20.);
IF TEMP=1 THEN
BEGIN
FOR M:=1 TO MAX DO
BEGIN
FLAG:=0;
FOR N:= 1 TO NUMZ  DO
FLAG:=FLAG+(Z(.N,M+9.)*Z(.N,20.));
ZF(.M+9.):=FLAG;
END;
END
ELSE
BEGIN
TEMPSTR := NUMZ ;
KVALUES;
END ;
FOR POS:=1 TO MAX DO
BEGIN
RESULT:=1;
FOR J:= 1 TO NUMZ DO
RESULT:=RESULT*(1+K1*Z(.J,20.)*Z(.J,POS+9.));
ZF(.POS+9.):=(RESULT-1)/K1;
END;
END;
BEGIN   (*  MAIN*)
FINDVALUES;
UNITY;
YSUTILITITY  ;
ZSUTILITY ;
WRITELN ;
WRITELN ;
WRITELN ;
WRITELN ;
WRITELN ;
K:=0;
FOR J:= 1 TO NUMBER DO
K := K+ STORE(.J.) ;
```

```
WRITELN(' VALUEX  OF  X  ::::');
WRITELN;
WRITE(' BEST             WORST             C.E');
WRITELN('              A              B              C');
FOR J := 1 TO K DO
BEGIN
WRITELN;
FOR I := 1 TO 3 DO
WRITE(' ',X(.J,I.):12);
FOR I:= 7 TO 9 DO
WRITE(X(.J,I.):12);
WRITELN;
END;
WRITELN;
WRITELN;
FOR I := 1 TO MAX DO
WRITE('   ALT#',I:2,'       ');
WRITELN('      K');
FOR J:= 1 TO K DO
BEGIN
FOR I := 10 TO (9+MAX) DO
WRITE(X(.J,I.):15);
WRITELN(X(.J,20.):15);
WRITELN;
END;
WRITELN;
WRITELN(' VALUES  OF  Y  ::::');
WRITE(' BEST             WORST             C.E');
WRITELN('              A              B              C');
FOR J:= 1 TO NUMBER DO
BEGIN
FOR I := 1 TO 3 DO
WRITE(' ',Y(.J,I.):12);
FOR I:= 7 TO 9 DO
WRITE(Y(.J,I.):12);
WRITELN;
END;
FOR I := 1 TO MAX DO
WRITE('   ALT#',I:2,'       ');
WRITELN('      K1             K');
FOR J:= 1 TO NUMBER DO
BEGIN
FOR I := 10 TO (9+MAX) DO
WRITE(' ',Y(.J,I.):15);
WRITELN(Y(.J,19.):15,Y(.J,20.):15);
END;
WRITELN;
WRITELN;
```

```
WRITELN(' VALUES  OF  Z  ::::');
WRITE(' BEST            WORST              C.E');
WRITELN('                 A                 B                 C');
FOR J:= 1 TO NUMZ DO
BEGIN
FOR I := 1 TO 3 DO
WRITE(' ',Z(.J,I.):12);
FOR I:= 7 TO 9 DO
WRITE(' ',Z(.J,I.):12);
WRITELN;
END;
FOR I := 1 TO MAX DO
WRITE('    ALT#',I:2,'         ');
WRITELN('       K1              K');
FOR J:= 1 TO NUMZ DO
BEGIN
FOR I := 10 TO (9+MAX) DO
WRITE(' ',Z(.J,I.):15);
WRITELN(Z(.J,19.):15,Z(.J,20.):15);
END;
FOR  I := 1 TO MAX DO
WRITE (' ',ZF(.I+9.));
END  .  (*  MAIN  *)
```

VALUEX OF X ::::

BEST	WORST	C.E	A	B	C
2.00000E+00	5.00000E+01	4.00000E+01	1.04623E+00	-4.05926E-02	6.49872E-02
2.00000E-01	1.00000E+01	8.00000E+00	1.04244E+00	-3.97542E-02	3.26660E-01
1.00000E+00	1.00000E+02	3.00000E+01	-1.79433E-01	1.20208E+00	-1.90201E-02
2.00000E-01	2.50000E+00	1.80000E+00	1.20983E+00	-1.80176E-01	7.61719E-01
3.00000E-01	3.00000E+00	2.50000E+00	1.14663E+00	-1.16679E-01	7.61719E-01
1.00000E+01	9.50000E+01	2.50000E+01	-2.27931E-02	1.60007E+00	-4.47510E-02
1.00000E+02	0.00000E+00	6.50000E+01	-3.85815E-01	3.85815E-01	1.27869E-02
1.00000E+02	0.00000E+00	6.50000E+01	-3.85815E-01	3.85815E-01	1.27869E-02
1.00000E+02	0.00000E+00	6.50000E+01	-3.85815E-01	3.85815E-01	1.27869E-02
2.00000E+00	1.00000E+01	4.00000E+00	-9.57247E-02	2.01544E+00	-3.04712E-01
5.86000E-01	0.00000E+00	2.00000E-01	6.11519E+00	-6.11519E+00	-3.04712E-01
1.00000E+01	1.00000E+02	2.00000E+01	-1.99120E-03	2.00013E+00	-6.91223E-02
1.00000E+00	5.00000E+01	5.00000E+00	-2.05665E-04	1.18941E+00	-1.73254E-01
2.00000E-01	3.00080E+00	1.00000E+00	-1.62370E-01	1.33784E+00	-7.02979E-01
0.00000E+00	2.50000E+01	5.00000E+00	-3.90735E-02	1.03907E+00	-1.31226E-01

0.00000E+00	5.00000E+00	1.00000E+00	-3.90488E-02	1.03905E+00	-6.56250E-01
1.00000E+02	0.00000E+00	7.00000E+01	-1.97870E-01	1.97870E-01	1.80069E-02
1.00000E+02	5.00000E+01	6.00000E+01	1.03904E+00	-2.76562E+01	-6.56311E-02
1.00000E+02	5.00000E+01	7.00000E+01	1.78344E+00	-4.05986E+00	-1.6452[illegible]-02
1.00000E+02	5.00000E+01	6.50000E+01	1.19776E+00	-7.25435E+00	-3.60229E-02
5.00000E-02	1.00000E-03	8.00000E-03	5.67032E+02	-5.67053E+02	-3.60229E-02
1.00000E+02	4.00000E+01	5.00000E+01	1.01765E+00	-1.51897E+01	-6.75781E-02
1.00000E+02	3.00000E+01	4.50000E+01	1.05183E+00	-3.82168E+00	-4.30054E-02
1.00000E+02	4.00000E+01	7.00000E+01	5.46183E+03	-5.46250E+03	-3.05176E-06
0.00000E+00	8.00000E+00	2.00000E+00	-9.57247E-02	1.09572E+00	-3.04712E-01
3.50000E+01	1.00000E+00	1.00000E+01	1.11998E+00	-1.19603E+00	-6.56982E-02
1.00000E+01	1.00000E-01	1.00000E+00	1.00049E+00	-1.08053E+00	-7.69580E-01
1.00000E+02	5.00000E+01	6.00000E+01	1.03904E+00	-2.76562E+01	-6.56311E-02
1.00000E+02	5.00000E+01	6.00000E+01	1.03904E+00	-2.76562E+01	-6.56311E-02
1.00000E+02	5.00000E+01	6.50000E+01	1.19776E+00	-7.25435E+00	-3.60229E-02
1.00000E+02	5.00000E+01	6.50000E+01	1.19776E+00	-7.25435E+00	-3.60229E-02
1.00000E+02	5.00000E+01	6.50000E+01	1.19776E+00	-7.25435E+00	-3.60229E-02
3.50000E+01	0.00000E+00	2.00000E+01	-1.27472E+00	1.27472E+00	1.65466E-02

1.00000E+02	5.00000E+01	5.50000E+01	1.00099E+00	-1.01380E+03	-1.38409E-01
5.00000E+00	3.50000E+01	2.00000E+01	-1.09222E+04	1.09233E+04	-3.05176E-06
5.00000E+00	2.50000E+01	1.00000E+01	-9.58477E-02	2.01508E+00	-1.21826E-
6.00000E+01	2.00000E+00	1.00000E+01	1.00696E+00	-1.19540E+00	-8.57727E-02
1.00000E+02	0.00000E+00	4.00000E+01	1.78323E+00	-1.78323E+00	-8.22754E-03
1.00000E+02	0.00000E+00	5.00000E+01	3.27730E+03	-3.27730E+03	-3.05176E-06
1.00000E+02	0.00000E+00	3.50000E+01	1.38549E+00	-1.38549E+00	-1.27930E-02
1.00000E+02	4.00000E+01	7.50000E+01	-1.02977E+00	6.55039E-01	1.13098E-02

ALT# 1	ALT# 2	ALT# 3	ALT# 4	K
9.62048518E-01	9.20321160E-01	9.55085019E-01	9.99777157E-01	5.00000000E-01
9.66506801E-01	9.33925230E-01	9.60982266E-01	9.97635229E-01	4.84239992E-01
8.08107638E-01	3.52537763E-02	7.93676007E-01	9.29363975E-01	3.13849495E-01
8.39370204E-01	6.92453534E-01	8.23896904E-01	9.60203318E-01	3.41554388E-01
8.02639175E-01	5.78889744E-01	7.80863504E-01	9.56363073E-01	3.50000000E-01
2.13410942E-03	2.13410942E-03	2.13410942E-03	1.90786197E-01	7.00000000E-01
1.00000000E+00	1.00000000E+00	5.58479306E-01	1.00000000E+00	1.03576135E-01
1.00000000E+00	1.00000000E+00	5.58479306E-01	6.87284511E-01	1.03576135E-01
8.73272128E-01	7.39994043E-01	6.87284511E-01	9.14171798E-01	1.03576135E-0
9.19629986E-01	9.44685385E-01	4.56287133E-01	1.00000000E+00	0.00000000E+00

9.42973861E-02	5.56464405E-02	1.86053626E-02	8.56177100E-01	0.00000000E+00
4.90483751E-01	4.52035387E-01	1.19938045E-02	4.73133409E-02	3.27017896E-01
7.36484141E-01	9.28193275E-01	1.77278619E-01	7.32618942E-02	5.50000000E-01
8.12663386E-01	2.09815181E-01	4.09097839E-01	5.00006179E-01	8.48908803E-03
2.40657606E-01	2.40657606E-01	2.40657606E-01	2.40657606E-01	5.00000000E-01
5.00002639E-01	5.00002639E-01	5.00002639E-01	5.00002639E-01	5.00000000E-01
2.08752941E-01	5.00042199E-01	5.00042199E-01	7.16464780E-01	0.00000000E+00
9.34560059E-01	8.93980950E-01	8.93980950E-01	9.84838083E-01	2.50000000E-01
9.32828831E-01	6.94743063E-01	6.01399711E-01	8.59898527E-01	2.09410179E-01
8.19541852E-01	7.91287070E-01	9.60970808E-01	9.14239019E-01	2.23495237E-01
1.32754627E-01	1.83805823E-01	1.83805823E-01	2.85894423E-01	2.40946801E-01
9.92906168E-01	8.83638236E-01	7.54243750E-01	9.92906168E-01	2.00000000E-01
9.87566995E-01	8.99954213E-01	8.99954213E-01	9.72151371E-01	1.93803460E-01
6.66687012E-01	5.00022888E-01	5.00022888E-01	5.83355586E-01	1.98581234E-01
1.00000000E+00	1.00000000E+00	1.80411242E-16	1.00000000E+00	1.76727647E-01
1.24913809E-01	1.82044827E-01	6.10858300E-01	7.76716790E-01	1.89893764E-01
8.93101650E-01	8.93101650E-01	9.93227327E-01	4.16701862E-01	2.43013652E-01
8.93980950E-01	8.93980950E-01	9.63787203E-01	6.50812391E-01	3.20000000E-[illegible]

9.34560059E-01	8.93980950E-01	8.93980950E-01	6.50812391E-01	2.86073904E 1
7.91287070E-01	6.15015976E-01	6.15015976E-01	5.00008359E-01	3.50000000E-01
8.58284491E-01	7.91287070E-01	8.19541852E-01	3.62303443E-01	2.15255592E-01
8.58284491E-01	7.91287070E-01	9.14239019E-01	7.11067390E-01	2.76950474E-01
3.59107105E-01	8.72236426E-02	1.33048709E-01	2.12677109E-02	5.50000000E-01
9.97043667E-01	9.85244324E-01	7.50190075E-01	9.99013833E-01	3.59206282E-01
9.99958802E-02	2.99990387E-01	3.66656036E-01	8.99995880E-01	5.00000000E-01
3.07705755E-01	3.71232206E-01	1.58161266E-01	8.74308735E-01	8.33301545E-02
9.36449226E-01	5.41633506E-01	1.58734808E-01	9.91901110E-01	1.66661580E-01
8.21132974E-01	6.94759684E-01	6.01418000E-01	9.32834131E-01	4.75399742E-01
6.00036621E-01	6.00036621E-01	6.00036621E-01	6.00036621E-01	3.21863739E-01
6.54673985E-01	4.41589629E-01	3.79240471E-01	6.06400097E-01	5.80000000E-01
6.83280369E-01	2.61377041E-01	1.23306073E-01	6.83280369E-01	0.00000000E+00

VALUES OF Y ::::

BEST	WORST	C.E	A	B	C
1.00000E+01	1.00000E+02	6.00000E+01	2.76749E+00	-1.68159E+00	4.98199E-[illegible]
1.00000E+01	1.00000E+02	5.00000E+01	-1.77221E+00	2.91350E+00	-4.97131E-[illegible]3
1.00000E+02	0.00000E+00	6.50000E+01	-3.85815E-01	3.85815E-01	1.27869E-02
2.00000E+00	1.00000E+01	4.00000E+00	-9.57247E-02	2.01544E+00	-3.04712E-01
5.86000E-01	0.00000E+00	2.00000E-01	6.11519E+00	-6.11519E+00	-3.04712E-01
1.00000E+02	0.00000E+00	1.00000E+01	1.00099E+00	-1.00099E+00	-6.92230E-02
0.00000E+00	1.00000E+01	2.00000E+00	-3.90686E-02	1.03907E+00	-3.28076E-01

1.00000E+02	0.00000E+00	7.00000E+01	-1.97870E-01	1.97870E-01	1.80069E-02
1.00000E+02	5.00000E+01	6.00000E+01	1.03904E+00	-2.76562E+01	-6.56311E-02
1.00000E+02	0.00000E+00	3.00000E+01	1.19773E+00	-1.19773E+00	-1.80130E-02
1.00000E+02	0.00000E+00	4.00000E+01	1.78323E+00	-1.78323E+00	-8.22754E-03
1.00000E+02	5.00000E+01	6.50000E+01	1.19776E+00	-7.25435E+00	-3.60229E-02
1.00000E+02	0.00000E+00	3.50000E+01	1.38549E+00	-1.38549E+00	-1.27930E-02
1.00000E+02	0.00000E+00	4.00000E+01	1.78323E+00	-1.78323E+00	-8.22754E-03
1.00000E+02	0.00000E+00	3.50000E+01	1.38549E+00	-1.38549E+00	-1.27930E-02
1.00000E+02	4.00000E+01	7.50000E+01	-1.02977E+00	6.55039E-01	1.13098E-02

ALT# 1	ALT# 2	ALT# 3	ALT# 4
9.64692010E-01	9.26867812E-01	9.58333020E-01	9.99767261E-01
8.21266525E-01	4.50192931E-01	8.03829530E-01	9.54407031E-01
2.98909404E-01	2.85122971E-01	1.88296286E-01	4.02624869E-01
9.19629986E-01	9.44685385E-01	4.56287133E-01	1.00000000E+00
9.42973861E-02	5.56464405E-02	1.86053626E-02	8.56177100E-01
6.14376094E-01	7.06727582E-01	1.05345187E-01	6.05345716E-02
3.70330123E-01	3.70330123E-01	3.70330123E-01	3.70330123E-01
2.08752941E-01	5.00042199E-01	5.00042199E-01	7.16464780E-01
6.78003072E-01	6.18819747E-01	6.38655976E-01	7.40853607E-01
7.44539357E-01	6.79987570E-01	5.52307975E-01	8.56858284E-01
8.93967087E-01	8.78608045E-01	9.40932292E-01	5.44182603E-01
8.07931378E-01	6.85986036E-01	7.37050546E-01	4.91855491E-01
5.88166717E-01	4.09682701E-01	3.51713508E-01	3.72478663E-01
2.31708398E-01	2.71198216E-01	2.22962139E-01	6.88158577E-01
9.63164427E-01	7.79511995E-01	6.98980256E-01	9.88266453E-01
6.83280369E-01	2.61377041E-01	1.23306073E-01	6.83280369E-01

K1	K
6.95000000E-02	4.05696080E-01
1.25000000E-04	5.50000000E-01
-6.25000000E-03	4.50000000E-01
0.00000000E+00	2.51315688E-01
0.00000000E+00	3.09278030E-01
6.07625000E-01	5.50000000E-01
0.00000000E+00	5.33259076E-01
0.00000000E+00	4.52996133E-01
2.30937500E-01	2.48508234E-01

1.07937500E-01	2.60000000E-01
5.98015625E-01	2.69072812E-01
6.43179687E-01	2.48508234E-01
4.59640625E-01	2.39652736E-01
-6.25000000E-05	2.30773030E-01
-1.44531250E-04	2.70000000E-01
0.00000000E+00	2.00457087E-01

VALUES OF Z ::::

BEST	WORST	C.E	A	B	C
1.00000E+01	1.00000E+02	8.00000E+01	1.06002E+00	-4.36228E-02	3.19046E-02
1.00000E+02	0.00000E+00	7.50000E+01	-9.58156E-02	9.58156E-02	2.43683E-02
0.00000E+00	1.00000E+02	7.50000E+01	1.09582E+00	-9.58156E-02	2.43683E-02
1.00000E+02	0.00000E+00	2.50000E+01	1.09575E+00	-1.09575E+00	-2.43744E-02
1.00000E+02	0.00000E+00	3.00000E+01	1.19773E+00	-1.19773E+00	-1.80130E-02

ALT# 1	ALT# 2	ALT# 3	ALT# 4
8.78457761E-01	6.42271507E-01	8.65307890E-01	9.73147445E-01
3.94789642E-01	3.82928810E-01	2.05159376E-01	6.97289269E-01
6.29949817E-01	8.12693522E-01	4.81937040E-01	5.55329461E-01
8.18260201E-01	7.51044251E-01	7.49685785E-01	6.92669330E-01
5.91440963E-01	4.23625174E-01	3.49181957E-01	6.51862222E-01

K1	K
2.00187500E-01	1.69804387E-01
-2.73437500E-05	2.02320841E-01
-2.73437500E-05	2.10701729E-01
-8.00781250E-05	2.20000000E-01
-8.00781250E-05	2.10701729E-01

6.672818970237292E-01

6.129905837134201E-01

5.290230874186095E-01

7.140798941467602E-01